Seasonal Adjustment

DANMARKS

Introduction

The aim of this paper is to provide a general description of the concept seasonal adjustment. The paper provides a thorough introduction to the subject aimed at statisticians involved in seasonal adjustment.

Furthermore, the paper describes the X-12 programme; that is the Seasonal Adjustment software used at Statistics Denmark.

The paper is edit by Head of section Christian Harhoff.

Statistics Denmark, January 2005

Peter Linde

0. Motivation

An important question

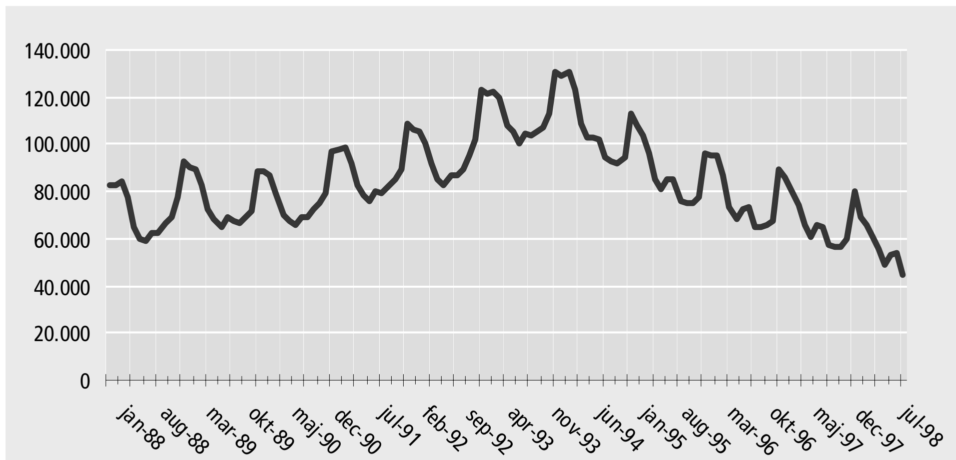
The most important question to pose when considering seasonal adjustment is to ask whether there is a need for seasonally adjusted figures.

In order to provide a suitable answer to this question it is necessary to realize the motivation behind the production of statistics. In Denmark a stable production of statistics is needed because the political decision-makers need information on the current state of society.

If for example an evaluation of unemployment figures is needed, and by looking at the figures one realizes that this task is difficult. The difficulty arises because regular seasonal patterns are present in the unemployment figures, which imply that a comparison between two consecutive months makes no sense. See the graph below.

Graph 0.1

Unemployed men in the period 1988-1998

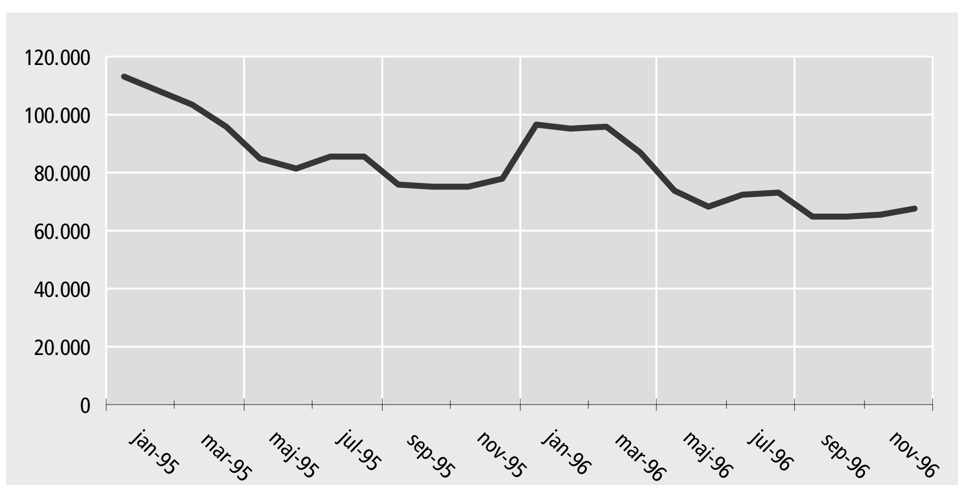


The graph shows that the figures vary regularly according to a stable pattern within every single year, i.e. a clear seasonal variation is present in the figures. Furthermore, it is seen from the graph that there is a variation in the level between years.

Using the graph given above, how does one for example evaluate the development in the figures between, e.g. December 1995 and January 1996? In order to fulfil this purpose a part of the figure is graphed below, covering only the years 1995-1996:

Graph 0.2

Unemployed men in the period 1995-1996



From the detailed graph above is seen a steep rise from December 1995 to January 1996, but from graph 0.1 showing the full period it appears that a rise always occurs between these two months. It is therefore necessary to determine how large a proportion of the rise that can be assigned to this normal seasonal variation. In order to perform the evaluation it is necessary to adjust for the part relating to the seasonal variation. The seasonal variation is hence considered a disturbing factor for which adjustments have to be made.

Seasonal adjustment is necessary

Consequently, it has to be concluded that there is a need for seasonally adjusted figures. Seasonally adjusted figures are the tool enabling the finding of turning points in the economical development. To the economic decision-makers, this information is crucial. The faster a turning point is determined the less exaggerated intervention in the economy is needed.

0.1 Time series analysis using component models

Time series can be analyzed by using the so-called component models. In a component model the time series is fragmented into four parts.

The definition of a component model

Considerations on how the problem of extracting the normal seasonal movements from the series is to be solved have resulted in an idea of using a so-called component model has evolved.

The four components

In the component model the time series is assumed to have been composed by four different components, they are;

- **The seasonal component S**, which is the cyclical periodical short-term movements having a length of less than one year[1].
- **The cyclical component C**, which is the cyclical movements having a longer periodicity than 1 year. A typical feature of these movements is that they are of a far smaller magnitude than the seasonal movements.
- **The trend component T** is the long-term movement in the data.
- **The irregular component I** contains everything else, but the three components given above.

Extreme observations

In the model, assumptions are explicitly expressed that the irregular movements are random. The irregular movements contain the so-called extreme observations.
Extreme observations are, for example, nature disasters or strikes.
It is assumed that these four components describe the series in consideration by either the sum or the product.

When considering an observed time series it is often difficult to distinguish between the mid-term fluctuations and the trend component, both unobserved series, these two components are often united into the term TC, which is called the trend-cycle.

0.1.1 The seasonal component

The purpose of analysing the component models is to eliminate the non-observed seasonal movements. Therefore, the following question needs to be answered:

How is the seasonal variation measured? If it is present how is it removed?

The seasonal component contains movements that are repeated in a more or less regular manner each year. In most series, a similar pattern is typically observed within a calendar year. The seasonal component contains typically, both the so-called climatic factors and the so-called institutional factors.

Climatic factor…

The climatic factor is a concept covering all normal weather-related conditions.

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[1] 1 Component models can be used on series that have shorter periodical movements, the duration of the movement could for example be 1 week or 24 hours, but this is not relevant for Statistics Denmark.

…and institutional factor

The institutional factor is a concept covering all calendar-related conventions to which society is adapted to. For example, it is seen that the concept covers the placement of religious holidays, the holiday season or the due dates of payments from/to public authorities.

0.2 Methods to eliminate the seasonal movements

There is no unambiguous way of eliminating the seasonal movements…

When the aim is to remove the seasonal component from a certain time series, then there is no unambiguous approach to take. This is obviously due to the fact that an unobserved element has to be removed.

…many possibilities exist

Many approaches of removing the seasonal variations can be taken. Very simple approaches, such as a simple calculation of growth rates can be used or very advanced model-based methods. Some of these methods will be demonstrated below.

0.2.1 Calculation of a yearly growth rate

The development between two years is calculated

The simplest approach possible in order to remove the normal seasonal movements is to calculate the development between two consecutive years, using the same period.

This method may work…

The simple approach described above works in an acceptable way, when the seasonal movements in the series in consideration are very regular, with respect to both the timeliness and the magnitude of the seasonal pattern.

…but it may not work

The method has certain disadvantages: It yields no information on the level of the original series, as well as no information on the development within a certain yearly period. If for example a rise of x% in the unemployment rate from July year t-1 to July year t has been found, then we could consider a number of quite different situations:
a. The unemployment has a steadily increasing long movement with a rise of x% per year.
b. There has been a rise in the preceding months, but it has been turned into a downward movement.
c. The unemployment has been constant within the first months or maybe even decreasing, but in the latest months a steep increase has occurred.

0.2.2 Advanced methods

When seasonal adjustment is performed far more advanced methods than the one described above are usually used.

The X-11 family or TRAMO/SEATS is applied

Most statistical agencies throughout the world apply either some members of the so-called X-11 family or they have changed the method to one with a model-based approach, for example TRAMO-SEATS.

Demetra

Eurostat has initiated the development of a user-friendly windows based surface to both X-12 and TRAMO/SEATS, this surface is called Demetra. Both the X-11 family, TRAMO/SEATS and Demetra will be described in this paper.

1. Definitions and concepts

This chapter gives an introduction to the basic concepts of undertaking seasonal adjustment.

1.1 Time series

A time series is simply time observations of the same concept. A certain concept is observed at regular intervals, using the same distance between intervals.

Example 1.1.1 different time series
The Gross National Product (GDP), quarterly
The consumer price index, monthly
Unemployment figures, monthly

1.2 Decomposition

The time series is decomposed…

The observed time series is denoted X_t. A time series is decomposed when the aim is to eliminate the seasonal variations.

…into unobserved components

When a time series is decomposed it is then split up into four non-observed components. A certain relation is assumed between these four components. The decomposition is either of the additive type (A) or the multiplicative type (M)[2].

Model definitions

The multiplicative model is defined by:

$$X_t = T_t C_t S_t I_t \qquad \text{(M)}$$

The additive model is defined by:

$$X_t = T_t + C_t + S_t + I_t \qquad \text{(A)}$$

T is the trend, C is the cycle, S is the seasonal component and I is the irregular component, as defined in section 0.1.

The decomposition of a given time series into the four components is a decomposition of an observed series into four non-observed series. This is the reason for the non-existence of theoretical guidelines, both with respect to the type of the decomposition or the definitions of the concepts seasonal component, the trend component and the cyclical component.

1.2.1 Example. An additive model[3]

An example of an additive composed time series is constructed in the table given below. The construction is simply a definition of the three subcomponents (no irregular component) generating the time series. The series is assumed to be observed quarterly over three years. The three terms defining the time series are given by;

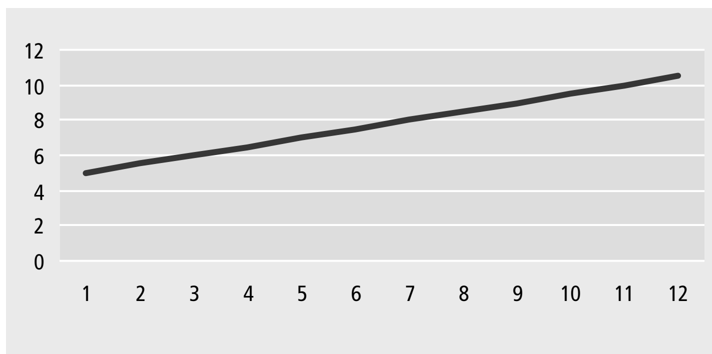
[2] These are the standard choices of models, but they do not cover all time series.

Table. 1.2.1.1 **Component series in an additive model**

Year	Quarter	Trend T	Cycle C	Season S
1	1	5,0	0,0	0,0
	2	5,5	0,1	0,4
	3	6,0	0,3	0,2
	4	6,5	0,4	-0,6
2	1	7,0	0,5	0,0
	2	7,5	0,4	0,4
	3	8,0	0,2	0,2
	4	8,5	-0,1	-0,6
3	1	9,0	-0,3	0,0
	2	9,5	-0,5	0,4
	3	10,0	-0,5	0,2
	4	10,5	-0,3	-0,6

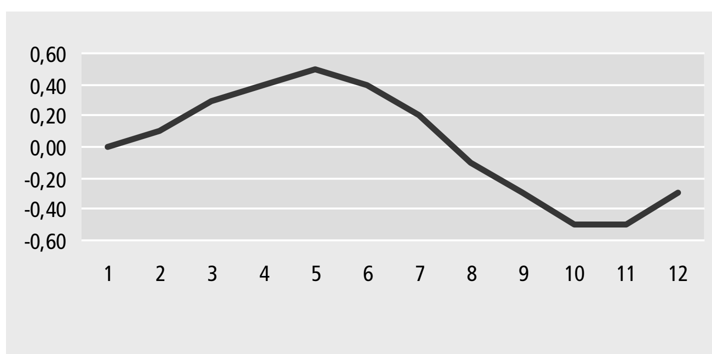
These three factors are given in figures below:

Graph 1.2.1.1 **Trend**



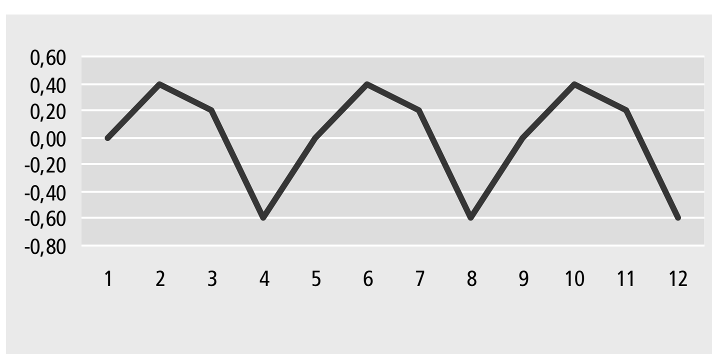
Cycle

Graph 1.2.1.2



Seasonal

Graph 1.2.1.3



These three components add up to the time series which is the only observed series

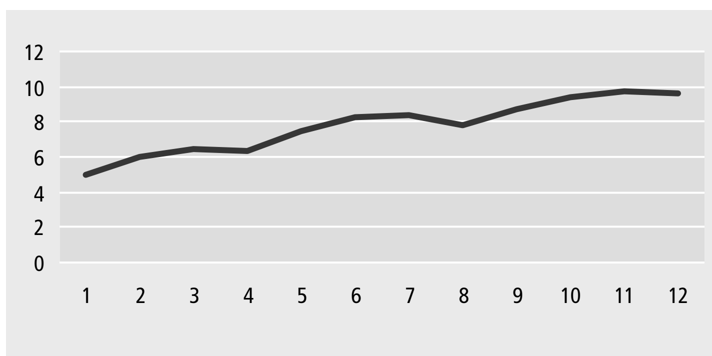
The time series X is defined as the sum of the three components given above:

Table 1.2.1.2 **A time series for an additive model**

Year	Quarter	Time series
		X=T+C+S
1	1	5
	2	6
	3	6,5
	4	6,3
2	1	7,5
	2	8,3
	3	8,4
	4	7,8
3	1	8,7
	2	9,4
	3	9,7
	4	9,6

The time series given graphically is the following:

Graph 1.2.1.4 **Time series**



1.2.2 Example. A multiplicative model[4]

Using the same approach as in the example of the additive model above, a multiplicative composed time series is constructed.

Therefore, the three components that multiply up to the time series are given below (no irregular component).

The series is observed quarterly over a period of three years. The three components that define the time series are given by:

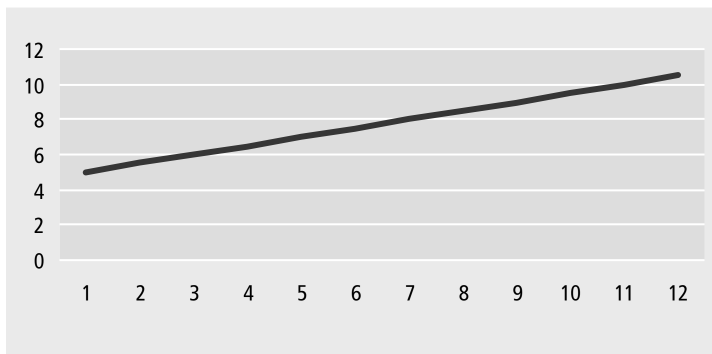
4 [illegible]

Table 1.2.2.1 **Component series in a multiplicative model**

Year	Quarter	Trend T	Cycle C	Seasonal S
1	1	5,0	1,00	1,00
	2	5,5	1,02	1,08
	3	6,0	1,05	1,04
	4	6,5	1,08	0,88
2	1	7,0	1,10	1,00
	2	7,5	1,08	1,08
	3	8,0	1,04	1,04
	4	8,5	0,98	0,88
3	1	9,0	0,93	1,00
	2	9,5	0,90	1,08
	3	10,0	0,90	1,04
	4	10,5	0,94	0,88

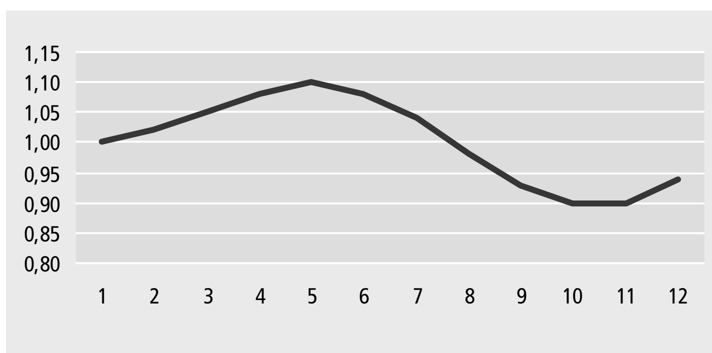
These three factors are given in figures below:

Graph 1.2.2.1 **Trend**



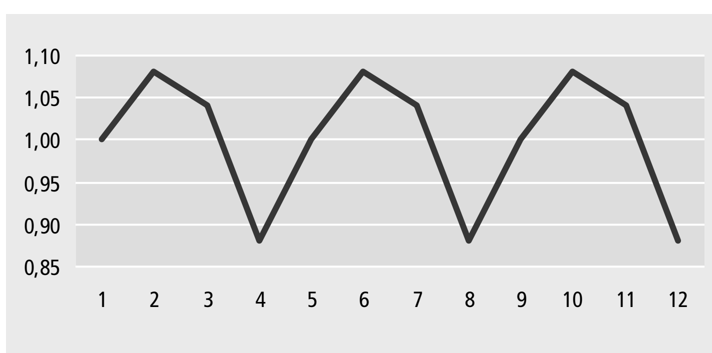
Cycle

Graph1.2.2.2



Seasonal

Graph 1.2.2.3



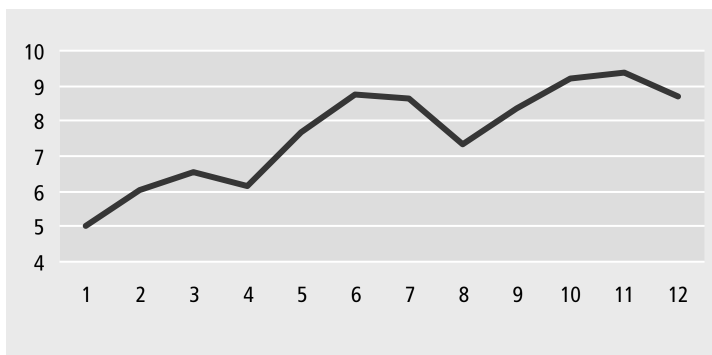
These three components add up to the time series which is the only observed series. The actual time series Y is defined as the product of the three components given above.

Table 1.2.2.2 **Time series from a multiplicative model**

Year	Quarter	Time series
		TCS
1	1	5,00
	2	6,06
	3	6,55
	4	6,18
2	1	7,70
	2	8,75
	3	8,65
	4	7,33
3	1	8,37
	2	9,23
	3	9,36
	4	8,69

The time series given graphically is the following:

Graph 1.2.2.4 **Time series**



1.2.3 Differences of the multiplicative and the additive models

Two different types of models have been described above, namely the additive model and the multiplicative model. For each time series considered it is therefore necessary to clarify which of the two models describe the data in the best way. There are some quite general differences between the structures in the two models, described below:

The additive model

In the additive model, the seasonal variation is independent of the absolute level of the time series, but it takes approximately the same magnitude each year.

The multiplicative model

In the multiplicative model, the seasonal variation takes the same relative magnitude each year. This means that the seasonal variation equals a certain percentage of the level of the time series. The amplitude of the seasonal factor varies with the level of the time series.

1.2.4 Joint treatment of the trend and the cycle

The distinction between the trend and the cycle is not obvious.
The trend is defined as a steady long-term movement. However, a clarification of the meaning long-term is needed.

If for example a decrease through several years is observed in a specific time series, then it is not obvious which of the two types of phenomena have to be observed. Is it a decreasing trend or is the pattern part of a cycle with a long duration?

In a typical analysis using component models these two series are treated jointly. The joint component is called the trend-cycle.

1.3 Types of seasonal variation

The seasonal pattern changes with respect to…

In economic time series covering longer periods it is often observed that the seasonal pattern changes.

...placement

Typical changes in the placement of the seasonal variation are seen when institutional changes, such as change of due dates in payments to or from the public authorities or changes in tax deduction rules.

Changes in the placement of the seasonal pattern can also be due to climatic factors. All seasons in Denmark are impossible to restrict to certain months. Such a climatic instability yields in itself a sliding pattern of the seasonal variation.

1.4 Moving averages

Centred moving averages

The seasonal adjustments that are performed in programmes of the so-called X-11 family are all based on centred moving averages. Moving averages in themselves have a smoothing effect on seasonal variations.

Moving averages are typically calculated using a certain observation as reference point. In seasonal adjustment the centred moving average is typically used.

Examples

A centred moving average with a length of 5, centred around the 7th observation, equals

$$\frac{1}{5}(x_5 + x_6 + x_7 + x_9 + x_{10})$$

A centred moving average with a length of 6, centred around the 7th observation, equals

$$\frac{1}{6}(\frac{1}{2} \cdot x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + \frac{1}{2} \cdot x_{10})$$

2. Requirements of the seasonal adjustment

As the concept seasonal variation is not theoretically defined it is not possible to express objective theoretical demands of the optimal seasonal adjustment method.

All suggestions of seasonal adjustment methods will therefore have to be tested empirically, and thereafter the final choice of model has to be based on subjective considerations, such as whether the results from the seasonal adjustment procedure are reasonable.

Internal working paper

In the report entitled 'Rapport vedr. sæsonkorrigering i Danmarks Statistik' written in 2001 by the working group determining the guidelines on seasonal adjustment at Statistics Denmark, a number of different demands to seasonal adjustment is discussed. The conclusions of this report are given below.

This working group is from now on called the working group.

2.1 Pre-adjustments

It might be necessary to pre-correct a series before the seasonal adjustment procedure itself is performed.

Seasonal adjustment removes the regular seasonal pattern

Seasonal adjustment is performed in order to remove the regular seasonal pattern from a series. Before this is performed, it is necessary to determine what is meant by the regular seasonal variations. This task cannot be performed if certain observations exist that blur the regular seasonal pattern.

Disturbing factors are to be removed

Before performing the seasonal adjustment procedure on a given series it has to be considered whether the series contains any untypical pattern that has to be taken into consideration in advance. Pre-corrections can be either permanent or temporary.

Permanent pre-adjustments

A permanent pre-adjustment is a correction performed before the seasonal adjustment is performed, then the pre-adjusted series is used as input series in the seasonal adjustment programme, such as X-12.

When permeate pre-adjustments have been performed then the final seasonally adjusted series is a series that is, both adjusted for normal seasonal variation and the known disturbing factors, which are removed in the pre-correction phase.

Temporary pre-adjustments

A temporary pre-adjustment removes the extreme values that might disturb the estimation of the normal seasonal pattern, but the pre-adjustment is removed in the final seasonally adjusted series. This series will therefore include the extremes from the original series.

2.1.1 Permanent pre-adjustments

Permanent pre-adjustments…

Permanent pre-adjustments occur quite generally.

...occur when a series has a break…

When a time series breaks then there is need for a permanent pre-adjustment. This happens regularly for index series when the base year is changed.

...when trading days has to be adjusted for....

Both the number of different weekdays and the total number of days vary from month to month. This fact will often disturb the development of monthly time series.

In statistics on production it is typically the number of monthly workdays that determines the development of the series. Statistics on turnovers are typically disturbed by both the number of trading days and the pattern of weekdays.

All seasonal adjustment programmes include a regression model, which automatically estimates a correction for the trading-day variation (trading-day adjustment).

..or when Easter has to be adjusted for.

In a number of different series the placement of Easter has an effect. This is due to the fact that Easter is a so-called moving holiday that can be in either March or April. Both monthly and quarterly series can therefore be affected by the placement of Easter.

All seasonal adjustment programmes include a regression model, which automatically estimates a correction for the placement of Easter.

2.1.2 Temporary pre-adjustments

In certain series, extreme non-typical events such as strikes or nature catastrophes can disturb the estimation of the normal seasonal pattern. In such occurrences temporary pre-adjustments are performed.

The seasonal component should not be affected

The seasonally adjusted series have to show the non-typical development, but this development should not affect the estimation of the seasonal component.

2.2 The irregular component

A seasonally adjusted series includes both the trend cycle and the irregular component. As mentioned earlier, it is implicitly in the assumption of a component model that the error term (the irregular component) is random.

A seasonal adjustment is not well performing when the oscillations of the irregular component exceeds the oscillations of the seasonal component.

In such a case, it is not easy for any seasonal adjustment package to recognize a stable seasonal pattern since the oscillations of the series are dominated by the movements of the irregular component.

2.3 Residual seasonal variation

Occasionally, one might in a seasonally adjusted series find systematic oscillations within a calendar year. Hence, there is a seasonal variation in the seasonally adjusted series. This phenomenon is denoted residual seasonal variation. The presence of this phenomenon indicates a change in the seasonal pattern over the years.

Residual seasonal variation is a problem…

Residual seasonal variation is a problem since the purpose of the seasonal adjustment exercise was to eliminate the seasonal variation of the original series.

…that has to be taken into account

The problem can be solved by adjusting on one or more of the possibilities available in the seasonal adjustment software used. A solution could be an adjustment of the length of the moving averages that are used in the seasonal adjustment procedure.

2.4 Direct or indirect seasonal adjustment?

The item of direct or indirect seasonal adjustment is relevant when a series, which equals a sum (or an average) of a number of component series, is considered. The difference in these two concepts is whether the aggregated series or the component series should be seasonally adjusted.

Direct seasonal adjustment

Direct seasonal adjustment of an aggregated series means that the series itself is seasonally adjusted. The procedure is here that first the series is added up and then it is seasonally adjusted.

Indirect seasonal adjustment

Indirect seasonal adjustment of an aggregated series means that the component series are seasonally adjusted and then added up. This sum is defined to be the indirect seasonal adjusted aggregated series. The procedure is here that first the component series are seasonally adjusted and thereafter they are added up.

Which method is the best?

Theoretically, there are no guidelines to which of the methods is the best. The choice of method should depend on the system of series that is considered.

Indirect seasonal adjustment is preferred at Statistics Denmark

Indirect seasonal adjustment is the method used at statistics Denmark. The method is used unless there are special conditions.

Indirect seasonal adjustment is the most intuitive concept…

Indirect seasonal adjustment seems the most intuitive easy concept to understand. The seasonal patterns in the component series could rather easily be imagined to cancel each other out or maybe worse add up to an incomprehensible seasonal pattern in the aggregated series.

...but should not always be used

A problem with indirect seasonal adjustment arises when different component series add up to the same total. An example is Export total. This could both be calculated as the sum of the export to all countries and as the sum of export of all goods. The two seasonally adjusted series obtained in these approaches are not identical.

2.5 Balancing the annual totals

When a series has been seasonally adjusted it rarely happens that the sum (or the average) over a calendar year of the seasonally adjusted series equals the sum (or the average) over the same calendar year of the original series. The difference can be removed by balancing the yearly totals.

Statistics Denmark balances the yearly totals

Statistics Denmark produces and publishes seasonally adjusted figures with balanced totals for all series that cover more than 5 years.

2.6. Uncertainties in seasonally adjustment

The resulting series of the seasonal adjustment changes back in time whenever a new observation prevails. This is simply due to the fact that X-12 uses long symmetric centred moving averages successively when estimating the trend and the seasonal factor.

Centred moving averages mean that one estimates the future…

As centred averages are also used for the newest observation then it is necessary to attempt a guess with respect to the future as a seasonally adjusted figure for the newest observation has to be calculated. The prediction of the future is done by the use of an ARIMA-model.

…this implies that seasonally adjusted figures are revised

The largest revision in the seasonal adjusted figures is therefore also seen in the latest observations. The only occurrence of no revision is the case where the models used have predicted the future correctly.

2.7 Revisions

As revisions prevail in seasonally adjusted figures, there may even be a need for large revisions, and consequently, it is necessary to determine a revision strategy.

Revisions of the former seasonally adjusted figure appear whenever a new observation is available. This is due to the fact that symmetric moving averages are applied in X-12.

The fact that the seasonally adjusted series changes even though the original futures do not change is obviously not clear to the non-technical user.

2.7.1 Optimal length of the revision period

On the one hand, it is obvious that the revision period should be short so that the users of the statistics are inconvenienced to the least possible.

On the other hand, it is clear that the period should not be too short, as this may make the development of the seasonally adjusted series unreliable.

These two contradictory demands do not yield a clear answer to what the optimal length of the revision period should be. A practical solution to these considerations is that the revision period should have a length so that seasonal adjustment with a new observation should only imply small revisions in that part of the series which is not in the revision period.

General revision strategy

The general revision strategy applied at Statistics Denmark is that all series should at least be revised 13 months/5 quarters back in time. At the most they should be revised 4 years back in time.

The demands on the minimum length of the revision period should always be fulfilled, while the demand on the maximum length can be deviated from, provided that the revisions further back in time are large.

Seasonally adjusted figures are finalized using yearly intervals.

2.8 Presentation of seasonally adjusted figures

Seasonally adjusted figures have been produced so that a comparison from period to period is possible. This makes a presentation of seasonally adjusted figures in a graph suitable.

Seasonal adjustment of a time series is not a simple transformation of the series, as for example the calculation of growth rates. This means that a presentation of seasonally adjusted figures has to be given, bearing this in mind, and that one has to be careful to give a thorough presentation.

2.8.1 The development of seasonally adjusted figures

Comments on seasonally adjusted figures should only be seen with regard to the development in relation to the previous period (or more previously seen together as for example a 3-month moving average).

A comment on the development of seasonally adjusted figures between the actual period and the same period a year before is meaningless. The reason for this is that seasonally adjusted figures are calculated so as to evaluate the development from one period to the next. Also, because the calculation of such growth rates in itself is a simple type of seasonal adjustment.

2.8.2 Interpretation of seasonally adjusted figures

When seasonally adjusted figures are given an interpretation it is important to remember that the seasonal adjustment procedure only removes the regular seasonal movements. If the sales of ice-cream, soft drinks or beer are extraordinarily large in a certain summer month, because the temperature of this month was extremely high, then this fact should be seen from the seasonally adjusted series.

The purpose of seasonal adjustment is the exact removal of the noise that is due to the regular seasonal movements, and not to smoothen the series as much as possible.

2.8.3 Monthly series

When comments from one period to the next are given, one should be cautious not to make too conclusive conclusions.

A change in the development can be only due to some randomness and cannot necessarily be taken as a sign of change of development. These problems occur especially for monthly series and especially for the latest observation available.

Generally, comments should instead be made with regard to comparisons based on longer periods, for example a comparison of the latest three months to the three months before.

Guidelines cannot be given with respect to the length used for the comparison periods. This is so because both the variability of the series and the uncertainty of the seasonal adjustment of the latest observation have to be taken into consideration.

Sometimes, if radical changes occur in seasonally adjusted series from one month to another, these changes are due to known circumstances as for example strikes political interferences or extremely good or bad weather.

2.8.4 The purpose of the analysis

The ultimate purpose of the analysis of conjecture series is typically a determination of the direction of the conjecture movement, as the aim is to determine the turning points.

Seasonal adjustment is the first phase of this determination, but since the seasonally adjusted series also contains a random element (the irregular component), the conjecture movement cannot be easily determined directly from the seasonally adjusted series.

There exists no known technique that solves this problem, and this is of course due to the simple fact that it is difficult to predict the economic development.

The analysis based on seasonally adjusted figures is the best empirical solution to this problem when the analysis is combined with a deep knowledge to all circumstances affecting the time series and the methodological part of the seasonal adjustment.

Seasonal adjustment is a supplementary tool that can lead the analyst to a clearer understanding than by considering the non-adjusted figures. But, unfortunately, this method cannot draw its own conclusions, and therefore it is necessary to use it with caution.

2.9 Manual seasonal adjustment

In order to show how seasonal adjustment is performed by the use of moving averages, as in X-12, we will below manually perform a simple seasonal adjustment of the series defined in the sections 1.2.1 and 1.2.2.

2.9.1 Seasonally adjustment of an additive series

In the additive mode it holds that X=T+C+S+I.

The seasonal adjustment is performed using five steps.

First step

In the first step four-quarter moving averages of the original observations are calculated.

This exercise eliminates the effect from the seasonal component under the assumption that the oscillations are regular.

The positive and the negative oscillations of the irregular component are also assumed to cancel out when averages are calculated, and hereby the effect of the irregular component is eliminated.

The moving average G provides therefore a first estimate of the trend and the cycle.

G=T+C

As symmetric moving averages are calculated, it is not possible to determine G for the two prior and the two last quarters.

Table 2.9.1.1 **Moving Averages in the additive model**

Year	Quarter	Time Series X	Moving Average G
1	1	5,0	…
	2	6,0	…
	3	6,5	6,26
	4	6,3	6,86
2	1	7,5	7,39
	2	8,3	7,81
	3	8,4	8,15
	4	7,8	8,44
3	1	8,7	8,74
	2	9,4	9,13
	3	9,7	…
	4	9,6	…

An example of the calculations done above is that for the 3rd quarter of the first year then:

$$G_{1,3} = \frac{\frac{1}{2}5{,}0 + 6{,}0 + 6{,}5 + 6{,}3 + \frac{1}{2}7{,}5}{4} = 6{,}26$$

Second step

The second step of the calculations is performed by giving a temporary estimate of the seasonal component plus the irregular component, by subtracting the moving averages from each quarter from the original observations.

This yields the differences D=X-G.

The difference is an estimate of the seasonal component where the irregular component I is included, since

D=X-G=(T+C+S+I)-(T+C)=S+I

Table 2.9.1.2 **Estimates of the seasonal component in the additive model**

Year	Quarter	Difference D=X-G
1	1	...
	2	...
	3	0,24
	4	-0,56
2	1	0,11
	2	0,49
	3	0,25
	4	-0,64
3	1	-0,04
	2	0,27
	3	…
	4	…

Third step Third step of the calculation is a summation over years of the different estimated (D) for the same quarter. This is simply done by calculating the averages S' over all years, one for each quarter.

This exercise minimizes the effect of the irregular component.

Table 2.9.1.3 **First estimate of the seasonal factors**

	D=X-G			Average
Quarter	Year 1	Year 2	Year 3	S'
1	...	0,11	-0,04	0,04
2	...	0,49	0,27	0,38
3	0,24	0,25	...	0,25
4	-0,56	-0,64	...	-0,60
Total				0,07

Fourth step

The final seasonal estimates have to sum up to 0. This is rarely the case for the seasonal estimates calculated in the third step.

It is therefore necessary to renormalize the seasonal estimates in order to fulfil this restriction. The renormalization is performed by a simple equal regulation of the four seasonal factors with the factor

$$\frac{1}{4}0{,}07 = 0{,}02\,.$$

The following relation therefore holds: $S = S' - \frac{1}{4}0{,}07$.

Table 2.9.1.4 **Final estimate of the seasonal factors**

	D=X-G			Average
Quarter	Year 1	Year 2	Year 3	S
1	...	0,11	-0,04	0,02
2	...	0,49	0,27	0,36
3	0,24	0,25	...	0,23
4	-0,56	-0,64	...	-0,62
Total				-0,01

A comparison of the seasonal factors estimated above with the original seasonal factors from this constructed time series shows that these two sets of seasonal factors are identical, when calculations are performed using 1 decimal. It should here be noted that such a comparison is not possible in the real world where the real seasonal factor is unknown.

Fifth step

When the seasonal figures have been calculated the seasonal adjustment is performed by a subtraction of the seasonal factors from the original data series.

The purpose of the seasonal adjustment exercise is to eliminate the effects from the regular seasonal variation, and hereby is obtained an estimate of the trend, plus the cycle where the irregular component is included;

$$Y\text{-}S=T+C+S+I\text{-}S=T+C+I$$

The seasonally adjusted figures are then

Table 2.9.1.5 **Seasonally adjusted figures in the additive model**

Year	Quarter	Observation X	Seasonal factor S	Seasonally adjusted series X-S
1	1	5,0	0,0	5,0
	2	6,0	0,4	5,6
	3	6,5	0,2	6,3
	4	6,3	-0,6	6,9
2	1	7,5	0,0	7,5
	2	8,3	0,4	7,9
	3	8,4	0,2	8,2
	4	7,8	-0,6	8,4
3	1	8,7	0,0	8,7
	2	9,4	0,4	9,0
	3	9,7	0,2	9,5
	4	9,6	-0,6	10,2

2.9.2 Seasonal adjustment of a multiplicative series

In the multiplicative model X=TCSI.

First step First the symmetric moving averages of the observations are calculated by G=TC as a first estimate over the trend and the cycle.

Table 2.9.2.1 **Moving averages in a multiplicative model**

Year	Quarter	Observation	4- quarters moving average G
1	1	5,0	...
	2	6,1	...
	3	6,6	6,31
	4	6,2	6,98
2	1	7,7	7,56
	2	8,7	7,96
	3	8,7	8,19
	4	7,3	8,34
3	1	8,4	8,49
	2	9,2	8,75
	3	9,4	...
	4	8,7	...

Second step T and C are then removed from the series, and this is done in a multiplicative model by division, i.e. the fraction Q is calculated

$Q = \frac{X}{G}$ is an estimate of the seasonal component

since $Q = \frac{X}{G} = \frac{TCSI}{TC} = SI$.

Table 2.9.2.2 **Estimates of the seasonal component in a multiplicative model**

Year	Quarter	fraction Q=X/G
1	1	...
	2	...
	3	1,05
	4	0,89
2	1	1,02
	2	1,09
	3	1,06
	4	0,88
3	1	0,99
	2	1,05
	3	…
	4	…

Third step The different seasonal estimates are joint, exactly as it was done in the additive model, by calculation of an average over the years, S'.

Table 2.9.2.3 **First estimate of the seasonal factors**

	Q=X/G			Average
Quarter	Year 1	Year 2	Year 3	S'
1	...	1,02	0,99	1,01
2	...	1,09	1,05	1,07
3	1,05	1,06	...	1,06
4	0,89	0,88	...	0,89
Sum				4,02

Fourth step The averages S' are normalized so that the calculated seasonal factors each varies around 1, i.e. they sum up to 4. The normalization is here performed by a multiplication of each seasonal factor with 4, and then dividing each factor with their sum.

$$S = \frac{4S'}{\sum S'}$$

Table 2.9.2.4 **Final estimate of the seasonal factors**

	Q=X/G			Normalized averages
Quarter	Year 1	Year 2	Year 3	S
1	...	1,02	0,99	1,00
2	...	1,09	1,05	1,07
3	1,05	1,06	...	1,05
4	0,89	0,88	...	0,88
Sum				4,00

Fifth step When the seasonal factors have been calculated, the seasonal adjustment is performed by dividing the original series with the seasonal factors. X/S=TCSI/S=TCI

Table 2.9.2.5 **Seasonally adjusted figures in a multiplicative model**

		Observation	Seasonal factor	Seasonally adjusted series
Year	Quarter	X	S	X/S
1	1	5,0	1,00	5,00
	2	6,1	1,07	5,70
	3	6,6	1,05	6,29
	4	6,2	0,88	7,05
2	1	7,7	1,00	7,70
	2	8,7	1,07	8,13
	3	8,7	1,05	8,29
	4	7,3	0,88	8,30
3	1	8,4	1,00	8,40
	2	9,2	1,07	8,60
	3	9,4	1,05	8,95
	4	8,7	0,88	9,89

3. Statistical analysis of time series models

A statistical analysis of time series models is, in principle, not different from the analysis of ordinary regression models.

Contents

A number of the general statistical concepts are shown in section 3.1, and in section 3.2 the autoregressive models are discussed. In section3.3 the MA-model and the ARMA-model is defined, in section 3.4 the ARIMA-model is defined, and in section 3.5 the ARIMA-seasonal model is defined. In section 3.6 the identification of ARIMA-models is described, and finally in section 3.7 is the control of ARIMA-models.

3.1 General statistical concepts

The concept statistical modelling covers a determination of functional relationship between one or more variables by using the information that lies in the observations available for the variables. It is not possible to give guidelines for the procedure of choosing the best model. Nevertheless, there is a number of quite general guidelines that the model should fulfil in order for it to be meaningful.

The model must describe the variation in the data. This demand seems needless and insignificant, but the best model has not always got the highest degree of explanation.

It is very important that the variables that are chosen as describing factors of the variable in consideration have a theoretical explanatory degree of explanation. The model used to describe data has to be meaningful in the sense that it must reflect reality.

The final model choice has to be based upon its theoretical characteristics. Even though these demands appear both simple and straightforward the process of determining the optimal model is both complicated and long.

The statistical models that are used in practice to describe the variation in a variable are often regression models.

The standard regressions model

In the standard regression model it is attempted to describe the variation in a variable (the dependent variable) by one or more independent variables (regressors). A regression model with one explanatory variable is

$$x_t = \alpha + \beta a_t + \varepsilon_t$$

Where x_t is the dependent variable, and a_t is the regressor α is the level, β is the slope and ε_t are the random elements of the model.

3.2 Autoregressive models

In regression models the values of the past of the dependent variable can be included as regressors, if this is applied the model is said to be autoregressive.

The simplest autoregressive model

The simplest autoregressive model is the so-called AR (1) model, defined by

$$x_t = \alpha + \beta x_{t-1} + \varepsilon_t \qquad \text{AR(1)}$$

This type of autoregressive model does not provide any definite explanation of the variation in the dependent variable. This model simply poses a claim of doing the same today as was done yesterday. Obviously, this assumption is reasonable when describing short-term economic behaviour.

An AR(p)-process

An autoregressive process of the order p (Without loss of generalization it is assumed that α=0) is defined by:

$$X_t = \gamma_1 X_{t-1} + \gamma_2 X_{t-2} + \ldots + \gamma_p X_{t-p} + \varepsilon_t \qquad \text{AR(p)}$$

Where ε_t is random variables that do not affect each other over time.

3.3 Definition of the MA-model and the ARMA-model

A MA(q)-model

Another type of model defines the dependent variable as a function of the random terms:

$$X_t = \varepsilon_t - \theta_1 \varepsilon_{t-1} - \ldots - \theta_q \varepsilon_{t-q} \qquad \text{MA(q)}$$

where $\theta_1, \ldots, \theta_q$ are parameters and the process $(\varepsilon_t)_{t \in N}$ is white noise.
The model is denoted a Moving Average model of the order q, a MA(q)-model.

The ARMA(p,q)-model

An ARMA-model is a mixed model. It is autoregressive of the order p, and it furthermore contains a moving average in the error terms of the order q.
An ARMA(p,q) model with mean 0 is the following:

$$x_t - \rho_1 x_{t-1} - \ldots - \rho_p x_{t-p} = \varepsilon_t - \alpha_1 \varepsilon_{t-1} - \ldots - \alpha_q \varepsilon_{t-q} \qquad \text{ARMA(p,q)}$$

The models can be described using the so-called lag operator L:

Let the lag operator be defined as $LX_t = X_{t-1}$.

The expression above can then easily be formulated using L:

$$(1 - \rho_1 L - \ldots - \rho_p L^p) x_t = (1 - \alpha_1 L - \ldots - \alpha_q L^q) \varepsilon_t$$

3.4 Definition of the ARIMA-model

The process X_t can be either stationary or non-stationary. The parameters in the given model can be restricted so that the process is stationary[5]. A non-stationary process is also denoted an integrated process.

Definition: An integrated process.

Define $\Delta = 1 - L$.
A process X_t is said to be integrated of order d, if $\Delta^d X_t = (1-L)^d X_t$ is stationary.

Example: A stationary process

$X_t = \varepsilon_t$, *where* ε_t *are normally independent identically normally distributed variables.*

Example: A non-stationary process

$X_t = \sum_{i=1}^{t} \varepsilon_i$ *is non-stationary. It is integrated by the order of 1, since* $\Delta X_t = X_t - X_{t-1} = \sum_{i=1}^{t} \varepsilon_i - \sum_{i=1}^{t-1} \varepsilon_i = \varepsilon_t$ *,is a stationary process.*

[5] An easy reference is 'Tidsrækkeanalyse for økonomer' by Anders Milhøj. A thoroughly more complicated reference is

An ARIMA(p,d,q) model for a time series $(x_t)_{t\in T}$ is a model where x_t is integrated by the order d, an the stationary series $(1-L)^d x_t$ is modelled by an ARMA(p,q) model, i.e. for this time series it holds that

$$(1-\rho_1 L - ... - \rho_p L^p)(1-L)^d x_t = (1-\alpha_1 L - ... - \alpha_q L^q)\varepsilon_t \quad \text{ARIMA(p,d,q)}$$

From this definition it is easily seen that the ARIMA-model is a generalization of all of the three models defined above, namely the AR, MA and ARMA-models;

An AR(p)-model is a ARIMA(p,0,0)- model, since ARIMA(p,0,0) is the same as

$(1-\rho_1 L - \cdots - \rho_p L^p)x_t = \varepsilon_t$, or

$$x_t = \rho_1 x_{t-1} + \cdots + \rho_p x_{t-p} + \varepsilon_t$$

A MA(q)-model is a ARIMA(0,0,q)-model, since ARIMA(0,0,q) is the same as

$$x_t = (1-\alpha_1 L - \cdots - \alpha_q L^q)\varepsilon_t = \varepsilon_t - \alpha_1 \varepsilon_1 - \cdots - \alpha_q \varepsilon_{t-q}$$

Finally, we see that an ARMA(p,q) model is a ARIMA(p,0,q)-model, since ARIMA(p,0,q) is the same as

$$(1-\rho_1 L - \cdots - \rho_p L^p)x_t = (1-\alpha_1 L - \cdots - \alpha_q L^q)\varepsilon_t$$

3.5 The ARIMA-seasonal model

The standard ARIMA-model that is defined as above does not include the regular seasonal variation pattern, and therefore it is necessary in this context to define the so-called ARIMA- seasonal model. In the ARIMA model, differences between two succeeding observations are included, while there are in the ARIMA-seasonal model differences between two observations that have exactly the seasonal length between them. The seasonal length is I 4 for quarterly figures and twelve for monthly figures.

An ARIMA-seasonal model is denoted ARIMA(P,D,Q) $_S$, where P is the order of auto regression in the seasonal model, D is the order of differencing, Q is the order of the moving average in the seasonal model and S is the seasonal length.

A seasonal-ARIMA-model

A seasonal-ARIMA(P,D,Q) $_S$ model is given by

$$(1-\beta_1 L^S - \cdots - \beta_P L^{SP})(1-L^S)^D x_t = (1-\phi_1 L^S - \cdots - \phi_Q L^{QS})\varepsilon_t$$

3.6 Multiplicative ARIMA-models

When the aim is to describe economic short-term series, then the two standard ARIMA models defined above are not adequate for a description on their own, Typically, there will both be trend- and seasonal movements present in a specific time series.

Definition

Such a combination of models is denoted an ARIMA (p,d,q)×ARIMA(P,D,Q) $_S$, written in formulas the following form is given:

$$(1-\rho_1 L-\cdots-\rho_p L^p)(1-\beta_1 L^S-\cdots-\beta_P L^{PS})(1-L)^d(1-L^S)^D x_t$$
$$=(1-\alpha_1 L-\cdots-\alpha_q L^q)(1-\phi_1 L^S-\cdots-\phi_Q L^{SQ})\varepsilon_t$$

Example: The Airline model
The so-called Box-Jenkins Airline model is the special case of the multiplicative ARIMA-model where it holds that p=P=0 and d=D=q=Q=1. This means that the Airline-model is the following:

$$(1-L)(1-L^S)x_t=(1-\alpha_1 L)(1-\phi_1 L^S)\varepsilon_t$$

This model is the ARIMA(0,1,1)×ARIMA(0,1,1) model.

3.7 Identification of ARIMA-models

When the ARIMA model has to be identified the first step is to determine how the time series should be transformed into a stationary series using differences. This order can be determined by the use of the so-called autocorrelation function.

Correlation

We know that it holds that the correlation between two variables x and y is given by $\frac{Cov(x,y)}{\sqrt{Var(x)Var(y)}}$, which is a number between -1 and 1.
If the autocorrelation function is graphed, it is denoted a correllogram.

Definition: Autocorrelation function

The autocorrelations function (ACF) is a similar function calculated for elements of a time series x_t and x_{t-j}.
The autocorrelation function (where n is the number of observations) is therefore estimated as

$$\rho(j)=\frac{n-1}{n-j-1}\frac{\sum_{i=j+1}^{n}(x_t-\bar{x})(x_{t-j}-\bar{x})}{\sum_{i=1}^{n}(x_t-\bar{x})^2}.$$

where $\bar{x}=\frac{1}{n}\sum_{t=1}^{n}x_t$

3.8 Controlling an ARIMA-model

First step in the model controlling phase is an examination of the estimated parameters of the model. It has to hold that the sum of the estimates in both the ordinary model and in the seasonal model each, seen numerically, has to be strictly less than 1. Empirically, this demand is often constrained to the sum being less than 0.9. If the sum of these parameter estimates exceeds this limit, this is equivalent to the fact that too many differences have been performed in the model.

The next step is an examination of the adequateness of the model in describing the variation of the data. The degree of explanation of the model is typically evaluated by a calculation of R^2. This is calculated by using the formulae

$$R^2=\frac{\hat{\sigma}^2-\hat{\sigma}^2_{res}}{\hat{\sigma}^2}=\frac{Variance\ in\ the\ original\ series-the\ residual\ Variance}{Variance\ in\ the\ original\ series}$$

R^2 takes values between 0 and 1. This calculated fraction often takes values close to 1. The fraction gives an expression of the part of the variation in the dependent variable that can be explained by the regression. By adding variables in the regression model, even variables with no a priori explanatory power, the fraction R^2 will always increase. This has to be taken into account when an interpretation of R^2 is given.

Another fundamental assumption in the model is that the residual term is a white noise process. This assumption can be examined by the use of the so-called Ljung-Box portmanteau test, which is a function of ACF defined as

$$Q_{LB} = n(n+2)\sum_{j=1}^{k} \frac{\rho_j^2}{n-j}$$

where ρ_j is the estimated residual autocorrelation for lag j, n is the number of residuals and k is the number of lags in the estimation.
Q_{LB} is asymptotically χ^2-distributed, the degrees of freedoms equalling k minus the number of estimated parameters in the model.

An estimated ARIMA-model is used for predictions. The ability of prediction of the model can be examined by comparing the values predicted by the model (on the basis of the observations in the prediction period) with the actual values in a previously defined period.

4. Method for seasonal adjustment

As the definition of the concept seasonal adjustment is very imprecise, there is no textbook answer with respect to the best seasonal adjustment method. There are many different approaches to seasonal adjustment in practice, and these range from simply drawing a graph to the advanced model based methods TRAMO/SEATS and X-12ARIMA. In this chapter, the methods will be presented in their historical order. In section 4.1, seasonal adjustment by the use of simple freehanded drawing is presented.

In section 4.2, the moving average method in seasonal adjustment is described, including the X-11 family. In section 4.3, the model based methods of seasonal adjustment are described, especially X-12 and TRAMO/SEATS.

4.1 Graphs

Seasonal adjustment is a task that was performed before the entry of the computer into the statisticians' world. Seasonal adjustment was then performed using figures only. First, the original series was drawn. Someone who possessed a detailed knowledge of the development of this specific series then drew an estimated stable trend-cycle curve. This curve was simply the seasonally adjusted series. This method appears very primitive today, but first of all, it has to be remembered that his freehanded drawing was performed by an expert, and secondly, it was the only possibility seen from a technological point of view.

The basis of the initial seasonal adjustment programmes was these freehand corrections of long time series.

Obviously, this method is highly problematic. It is never ever possible, not even for the same person to create precisely the same seasonally adjusted series. Furthermore, the method is very resource-demanding.

4.2The moving average method

Moving averages are often used when one assumes a component model relation for a time series. The application of moving averages enables in a simple manner the estimation of every component in the model. The moving averages are extensively applied in the X-11 family.

The X-11 family is a seasonal adjustment package that is widely used by statistical agencies all over the world. But on the other hand, it seems as if X-11 is one of the most criticized programmes in academia. This criticism is due to the fact that the method is based on different types of moving averages only. There is no explicitly defined statistical model, and the method is developed on an empirical basis.

4.2.1 History of the X-11 family

- In 1954 the so-called Census I method was invented by the US Bureau of the Census, and this step has without doubt been the most important in the history of seasonal adjustment.
- In 1955 this method was developed into Census II. Census II was essentially seen an electronic version of the manual methods that were previously applied in the seasonal adjustment world. The method was also denoted X-0 (experimental). The method was criticized in different aspects. Among other things, it was problematic that the basis of the method was purely empirical not having any fundament in theoretical statistics. Furthermore, it was found that Census II simply removed the large seasonal movements leaving the smaller movements untouched. This implies a disturbance in the relation between the components of the model. Sometimes Census II overestimated the seasonal effect.
- The problems described above led to the development of the versions X-3 up to X-10.
- Further development led in 1965 to the development of the so-called X-11 variant.
- In 1988 Statistics Canada developed an extended version of the X-11 method, the so-called X-11ARIMA version. The method uses an ARIMA (Auto Regressive Moving Average) model for prediction of both the future and the past lying before the start of the series. This is necessary as the seasonal adjustment method is based upon symmetric moving averages. The original X-11 programme extrapolated these out-of-sample observations rather randomly.
- A new version, the so-called X-12 ARIMA programme is developed by the US Bureau of the Census. It has to be noticed that even though it is a development of X-11, the approach behind X-12 is based on a model.

4.2.2 Description of the methods used in the X-11 family

The moving average method that is applied in the X-11 family can be described in main features by the following iterative process;

The first task is to include pre-adjustments if applicable.

The next step is the estimation of the trend-cycle of the pre-adjusted series. This is conducted by a calculation of centred moving averages. When the figures are quarterly then the length of the moving averages applied is four quarters, while the length is twelve for monthly series.

The trend-cycle is removed from the original series by division/subtraction (correspondence to the type of model, either multiplicative or additive) of the estimated trend-cycle in/from the original series. The series that have the trend-cycle removed are denoted the SI-rate (Seasonal and Irregular).

Extreme values of these SI-rates are substituted by less extreme values.

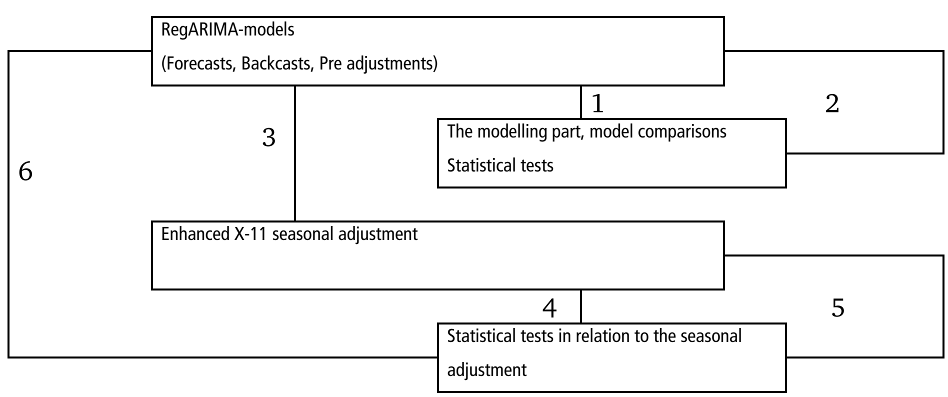
These modified values of the SI-rates are then smoothened by applying moving averages. The seasonal factors that are calculated in this manner are then divided up into or subtracted from the original series, and hence a first estimate of the seasonally adjusted series is obtained.

Using this initial estimate of the seasonally adjusted series as a starting point, the first steps of calculations (1.-5.) are repeated three times. The aim is an improvement, i.e. a smoothing of the trend-cycle estimate, and hence a better identification of extreme values.

4.3 The model based methods

4.3.1 X-12 Arima

The structure in X-12 Regarima is given in the following figure.



The figure is read as follows: The Reg-ARIMA-models are used as a starting point, then the next item is the model comparisons, and thereafter the aim is again the Regalia-part.

Has the modelling part turned out successfully in the sense that a suitable model has been chosen then the next step is to perform the seasonal adjustment. If no suitable model has been chosen, then one returns to the starting point until a suitable model has been found.

When the seasonal adjustment has taken place then the test statistics calculated in the seasonal adjustment procedure are considered. If the test statistics are acceptable, then the process is terminated. If this is not the case, another seasonal adjustment is performed.

4.3.1.1 The Reg-ARIMA-model

A Reg-ARIMA-model is an extension of the class ARIMA-models. These are characterized by a linear regression equation for the process x_t in the following equation

$$x_t = \sum_i \beta_i y_{it} + z_t$$

Where the y's are regression variables observed simultaneously with x, and the β's are the regression parameters. The error term z is assumed given by a general $\text{ARIMA}(p,d,q)\times\text{ARIMA}(P,D,Q)^s$ model.

This archetypical regression model is denoted a Reg-ARIMA-model.
In this model, it is implicitly assumed that the explanatory variables only affect the dependent variable x through present values, there are no historical values of the y's in this model formulation.

The extension in this model compared to the standard ARIMA-model is obvious. In this model is both the possibility of a standard regression equation in the mean of the process, and furthermore an extension of the standard regression model so that the error terms can be assumed to follow a general ARIMA-process. This set of models is fairly wide.

The specification of a Reg-ARIMA-model demands that both the regression variables (the y_{it}'s) and the ARIMA-model for the regression error terms z_t are defined.

4.3.1.2 Standard regression variables in X-12

Several standard regression variables have been included in X-12. See the X-12-ARIMA Reference Manual from the US Bureau of the Census for further explanations.

The simplest regression model is simply a constant term. When no differences are present in the Reg-ARIMA-model, then the constant term is simply the level in the regression. If differences are present in the model definition then X-12 ARIMA uses a trend as regression variable. The trend is defined in such a way that a differencing of this term according to the Reg-ARIMA-model yields a column, which is a constant term.

Other types of standard regression variables are the standard seasonal effects, such as trading day effects, holiday effects and several dummies used for modelling temporary movements of the series.

4.3.1.3 Monthly series

In a monthly series the standard seasonal effects are modelled by using 12 indicator variables, one for each month. Furthermore, if one also wants to include a constant term, then these 13 variables together are an over parameterization of the model. This is the reason for choosing 11 indicator variables and the constant term.

Trading day effects are present in a series when the series is affected by the number of different days in a calendar month. Trading day effects are modelled by variables given by (The number of Mondays),…,(The number of Sundays). Only six of these dummies are used since a constant term also is included in the model.

A working day effect is another approach to be used when the aim is to model differences in the days of the week. A working day correction is used on series where there is an effect on the series of the number of work days in a month, for example production series. A working day effect is modelled by the two variables (The working days) and (The non-working days). Only one of these is used, since a constant term is also included in the model.

Holiday effects can be seen in monthly series and the effect is due to holiday periods where the dates vary over time. The Easter effect is the most frequently found holiday covered by this concept.

X-12 can also take sudden level changes of a series into account, they can be either temporary or permanent. The following four types of regression variables are used in X-12, if necessary.

- AO(Additive Outliers): A constant is added at one given point of time.
- LS(Level Shift): A constant is added from a given point of time.
- TC(Temporary Change): A level shift with exponential growth is used.
- Ramp:
- A linear function in t (over a limited period of time) is added. The function equals -1 until the starting level. Then a linear function in t is used, until the end level, and thereafter the function equals 0.

4.3.2 TRAMO/SEATS -the ARIMA based model.

In TRAMO/SEATS, the same initial model is used as in X-12, the Reg-ARIMA-model

$$x_t = \sum_i \beta_i y_{it} + z_t$$

Where the first term is the deterministic regression variables, and the error term is assumed to follow the ARIMA(p,d,q)×ARIMA(P,D,Q) ^{s} process.
Exactly as in X-12, trading day corrections and Easter corrections and corrections for either temporary or permanent level shifts can be used as deterministic regressors.
In the procedure, test statistics are calculated and outliers are determined and corrected. If missing observations are present, interpolation is performed, and forecasts are calculated.

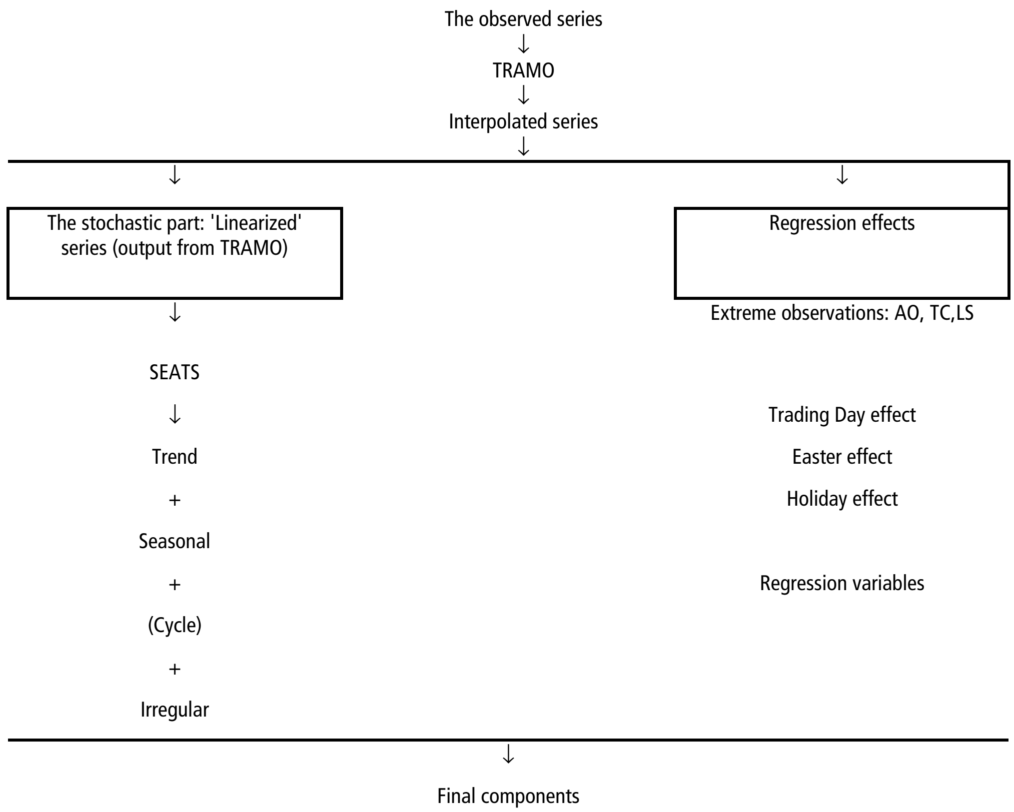
If the series that are to be seasonally adjusted have outliers, extreme observations or are affected by deterministic regressors, then the series have to be 'cleaned' by TRAMO.

The 'clean' and linearized series (output from the ARIMA-model) are decomposed in SEATS into the four standard components, namely the seasonal, the trend the cycle and the irregular component.

The big advantage of SEATS compared to the ad-hoc based methods is that it calculates optimal estimates and forecasts in the framework of well-defined statistical models and well-defined estimation criteria.

Graph 4.3.2.1

Overview of TRAMO/SEATS



4.3.3 Demetra

Demetra is a windows-based user surface for the two programmes X-12 and TRAMO/SEATS. The initiative of the development of Demetra was taken by Eurostat. Demetra is a very user-friendly surface, which is well-suited for the adjustment of large scale series. Demetra is used at Statistics Denmark for seasonal adjustment.

4.3.3 Different test statistics

When a seasonal adjustment is performed using X12-Arima in Demetra, different test statistics are calculated in order to examine the overall quality of the adjustment. The essential ones are described below.

- Ljung-Box test on residuals: This is a test for the presence of autocorrelations in the fitted residuals (of the chosen ARIMA model).
- Kurtotis: This is an indication of the presence of Kurtotis (4th central moment) in the fitted residuals (of the chosen ARIMA model).
- Forecast error over last year: This is an indication of whether the forecasted values vary too much around the true values. If this is the case, the fitted ARIMA model cannot fit the time series well.
- Percentage of outliers: This is an indicator of the number of outliers.
- Q-test statistic: This test statistic is given only for the X-11 type of seasonal adjustments. It is described in detail in appendix 2 below.

6. Applications at Statistics Denmark

In this chapter, a number of applications of the seasonal adjustment procedure in Statistics Denmark and a subset of the series that are published seasonally adjusted are shown. All series are adjusted using the X-12 procedure. In this chapter, graphical presentations are primarily given of the seasonal adjustment that is performed.

The series shown in this chapter are divided into four groups. The first group covers seasonal adjustments that are without any problems, given in section 6.1.1. The second group consists of series that have large irregular components seen in relation to the seasonal component. These are covered in section 6.1.2. The third group consists of series that need further examination. These are covered in section 6.1.3. The fourth group is a number of aggregate series where the difference between directly and indirect seasonal adjustment is considered. This is done in section 6.1.4.

In appendix 3, the details of the seasonal adjustments, for example the choice of model and tests statistics are given.

For each seasonal adjustment performed in the sections 6.1.1-6.1.3 two graphs are shown. The first one shows the original series and the seasonally adjusted series. The second shows the series of estimated seasonal factors and the series of estimated irregular factors.

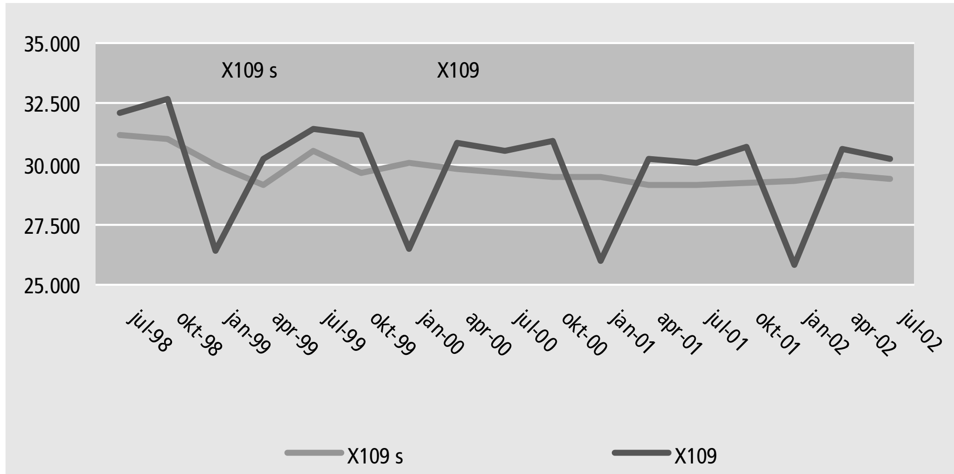
The first graph is self-explanatory. The second graph can be used for observing two different aspects of the seasonal adjustment. First of all, an evaluation of the regularity of the seasonal component can be performed, and secondly an evaluation of the relative size of the irregular component with respect to the seasonal component.

The aggregate series shown in section 6.1.4 are given graphically in one figure. This graph shows three series, namely the original series versus the direct seasonal adjusted series and the indirect seasonal adjusted series.

6.1.1 Non-problematic seasonal adjustments

6.1.1.1 ATP-employment: Agriculture

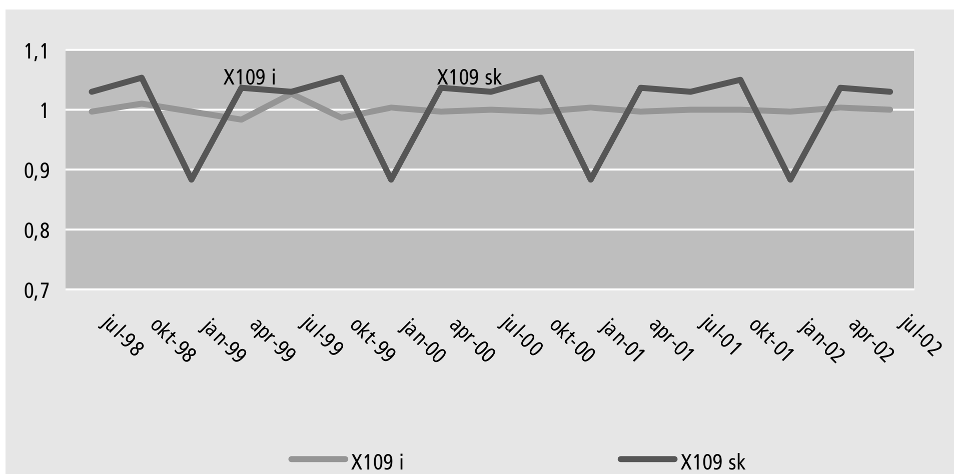
Graph 6.1.1.1.1



The result from the seasonal adjustment procedure shows that the series is indeed suitable for seasonal adjustment. Several test statistics have been calculated, indicating the suitability of seasonal adjustment. Not all possible test statistics have been calculated because the series is so short. See table A.3.2 below for details.

The series is quarterly and is observed from 3q 1998 to 3q 2002, a multiplicative model has been chosen, and the possibility of an Easter effect has been tested. This effect is not present in the series. The trading day effect has not been tested, as this effect is irrelevant for the employment series.

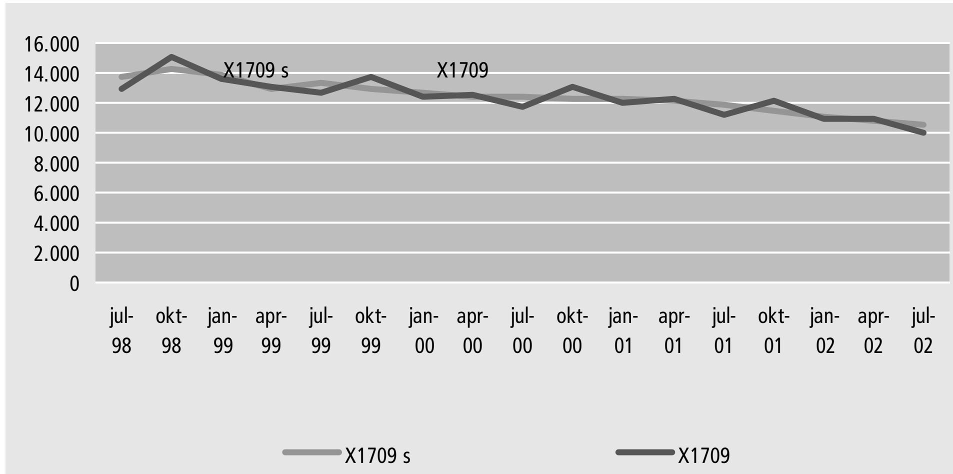
Graph 6.1.1.1.2



From the graph above, it is both seen that the estimated seasonal factor is stable over time, and that the irregular factor is small seen in relation to the seasonal factor. This means that the series contains a strong seasonal movement.

6.1.1.2 ATP-employment: Construction

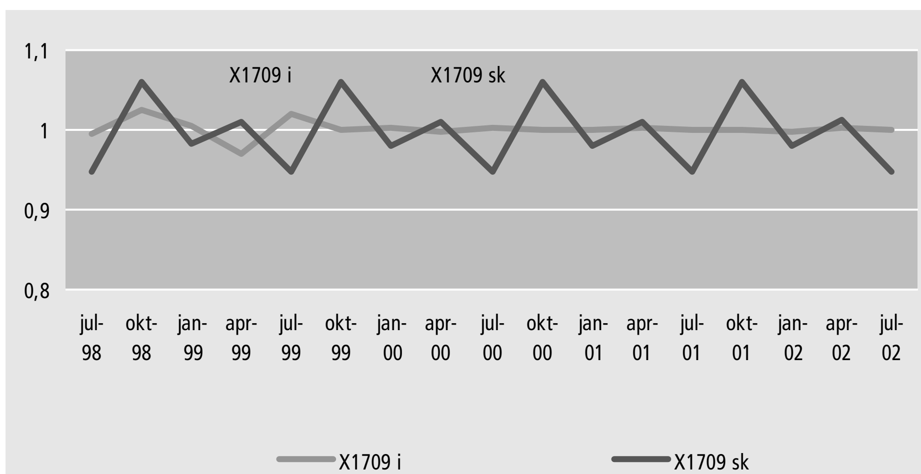
Graph 6.1.1.2.1



The result from the seasonal adjustment procedure shows that the series is indeed suitable for seasonal adjustment. Several test statistics have been calculated, indicating the suitability of seasonal adjustment. Not all possible test statistics have been calculated because the series is so short. See table A.3.2 below for details.

The series is quarterly and is observed from 3q 1998 to 3q 2002, a multiplicative model has been chosen, and the possibility of an Easter effect has been tested. This effect is not present in the series. The trading day effect has not been tested, as this effect is irrelevant for the employment series.

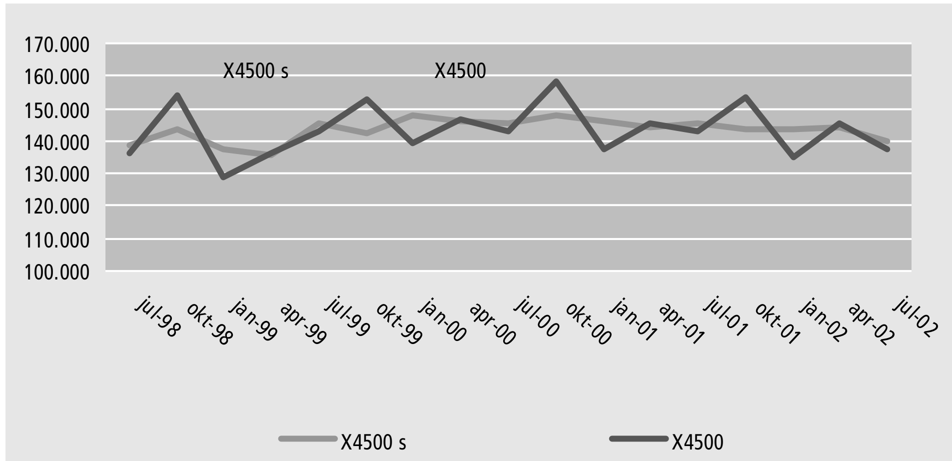
Graph 6.1.1.2.2



From the graph above, it is both seen that the estimated seasonal factor is stable over time, and that the irregular factor is small seen in relation to the seasonal factor. This means that the series contains a strong seasonal movement.

6.1.1.3 ATP-employment: Construction

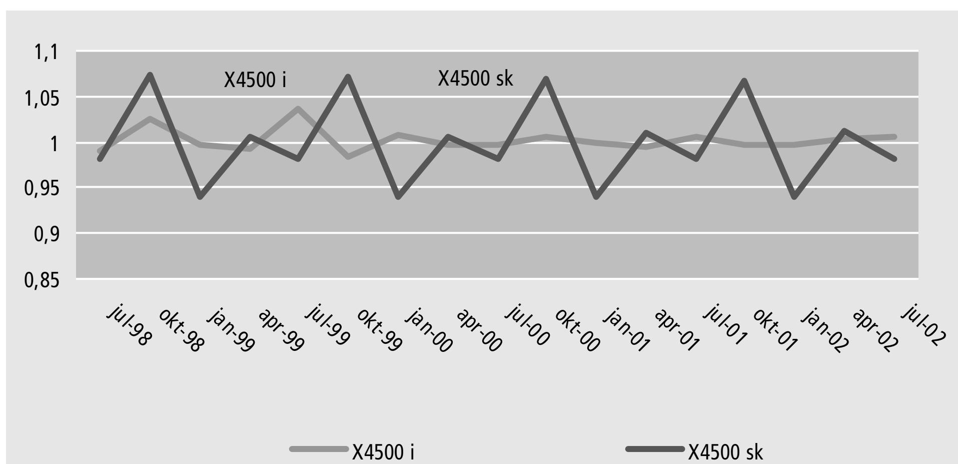
Graph 6.1.1.3.1



The result from the seasonal adjustment procedure shows that the series is indeed suitable for seasonal adjustment. Several test statistics have bee calculated, indicating the suitability of seasonal adjustment. Not all possible test statistics have been calculated because the series is so short. See table A.3.2 below for details.

The series is quarterly and is observed from 3q 1998 to 3q 2002, a multiplicative model has been chosen, and the possibility of an Easter effect has been tested. This effect is not present in the series. The trading day effect has not been tested, as this effect is irrelevant for the employment series.

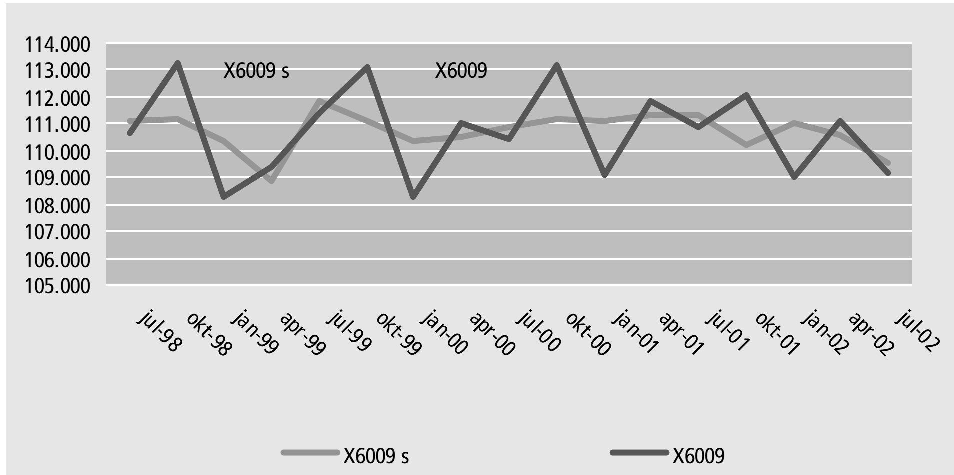
Graph 6.1.1.3.2



From the graph above, it is both seen that the estimated seasonal factor is stable over time, and that the irregular factor is small seen in relation to the seasonal factor. This means that the series contains a strong seasonal movement.

6.1.1.4 ATP-employment: Transport, mail and telecommunication

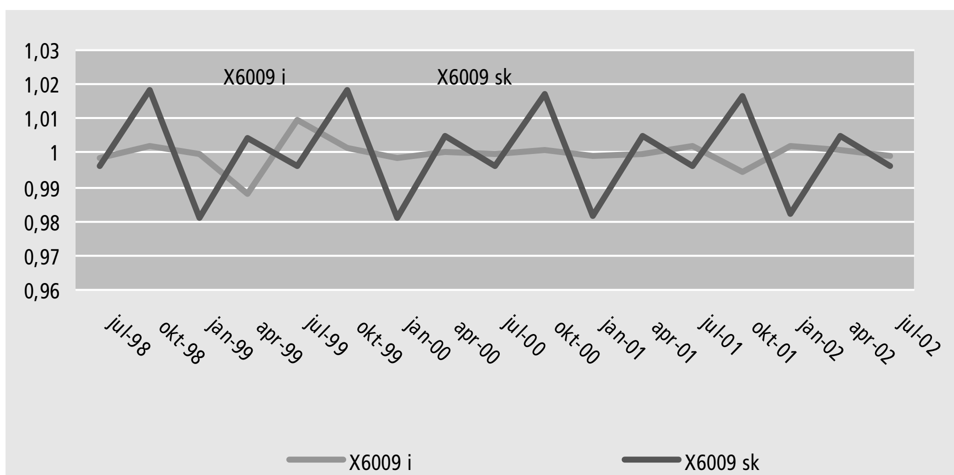
Graph 6.1.1.4.1



The result from the seasonal adjustment procedure shows that the series is indeed suitable for seasonal adjustment. Several test statistics have been calculated, indicating the suitability of seasonal adjustment. Not all possible test statistics have been calculated because the series is so short. See table A.3.2 below for details.

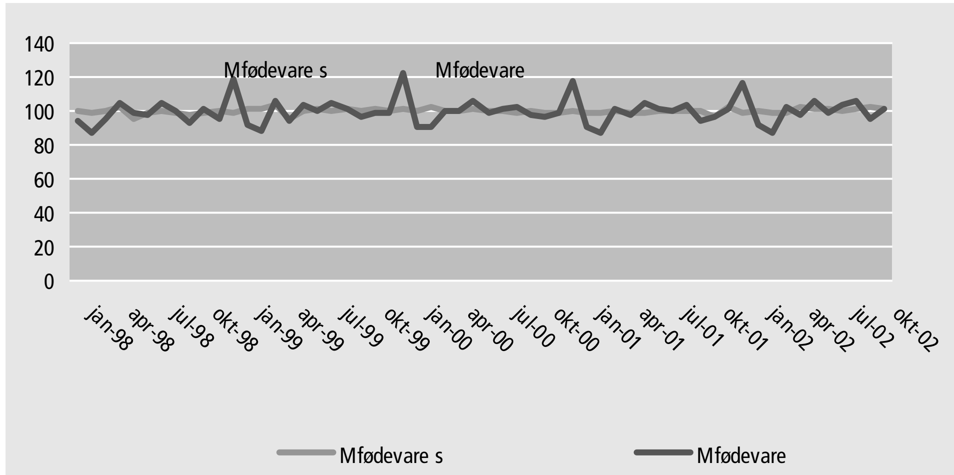
The series is quarterly and is observed from 3q 1998 to 3q 2002, a multiplicative model has been chosen, and the possibility of an Easter effect has been tested. This effect is not present in the series. The trading day effect has not been tested, as this effect is irrelevant for the employment series.

Graph 6.1.1.4.2



From the graph above, it is both seen that the estimated seasonal factor is stable over time, and that the irregular factor is small seen in relation to the seasonal factor. This means that the series contains a strong seasonal movement.

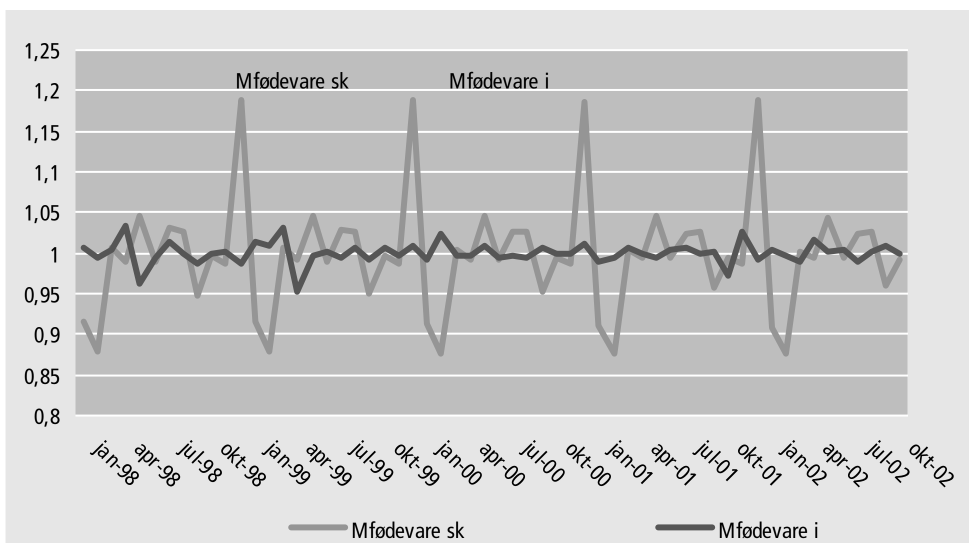
Graph 6.1.1.5.1



The result from the seasonal adjustment procedure shows that the series is indeed suitable for seasonal adjustment. Several test statistics have been tested, indicating the suitability of seasonal adjustment. All of these show that the series is performing well.

The series is monthly and is observed from January 1998 to October 2002, a multiplicative model has been chosen, and the possibility of an Easter effect has been tested. This effect is present in the series. The trading day effect has been tested as well and it is significant.

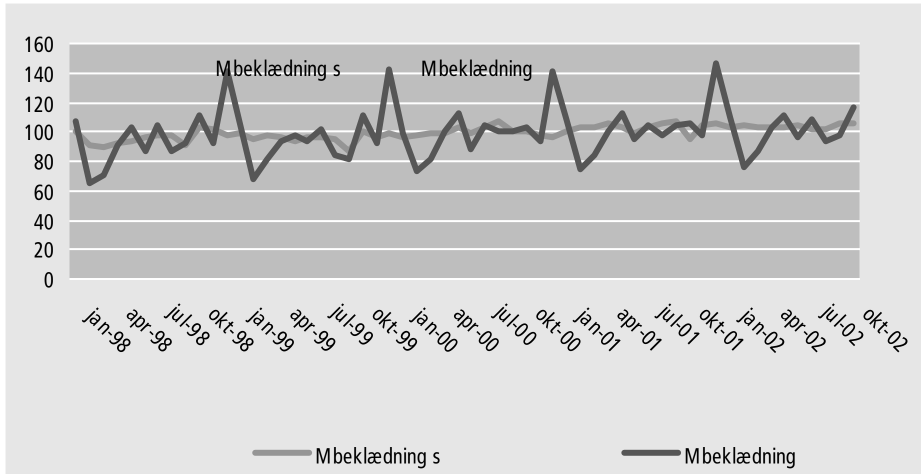
Graph 6.1.1.5.2



From the graph above, it is both seen that the estimated seasonal factor is stable over time, and that the irregular factor is small seen in relation to the seasonal factor. This means that the series contains a strong seasonal movement.

6.1.1.6 Retail index: Clothes

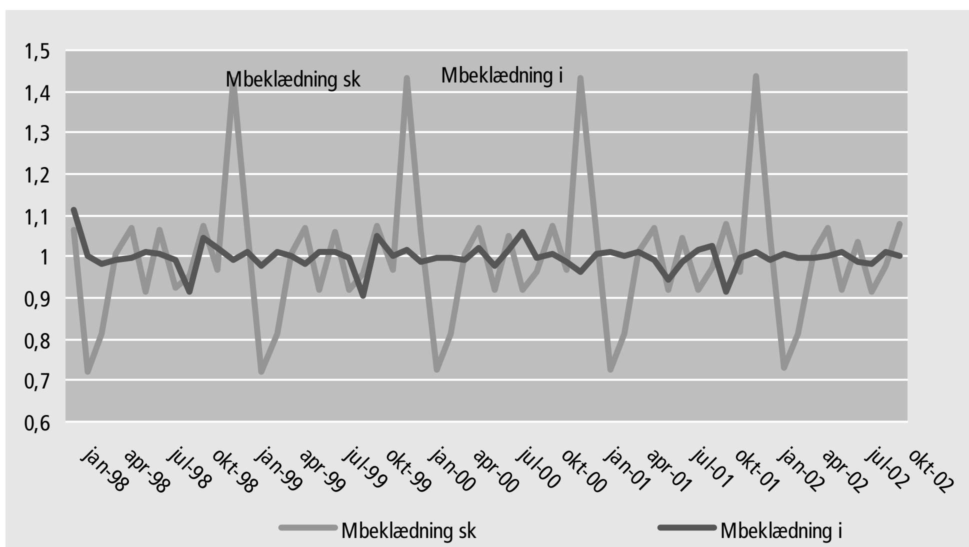
Graph 6.1.1.6.1



The result from the seasonal adjustment procedure shows that the series is indeed suitable for seasonal adjustment. Several test statistics have been tested, indicating the suitability of seasonal adjustment. All of these show that the series is performing well.

The series is monthly and is observed from January 1998 to October 2002, a multiplicative model has been chosen, and the possibility of an Easter effect has been tested. This effect is not present in the series. The trading day effect has been tested as well and it is significant.

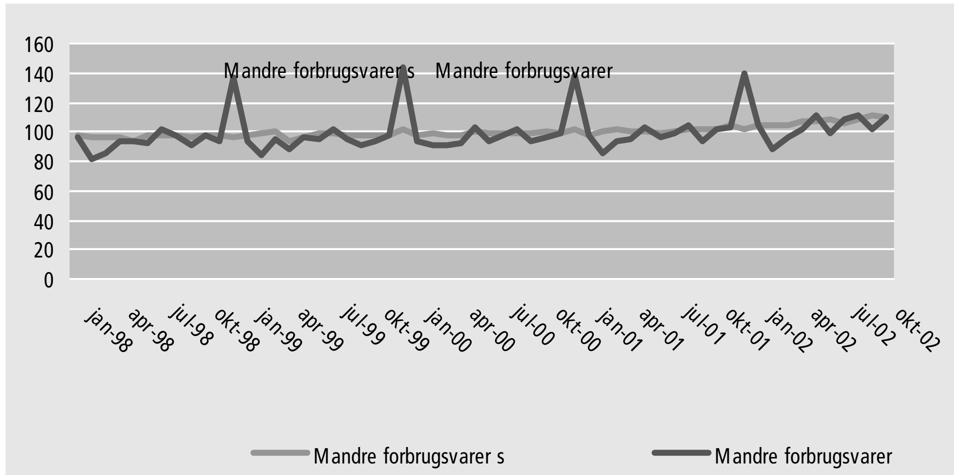
Graph 6.1.1.6.2



From the graph above, it is both seen that the estimated seasonal factor is stable over time, and that the irregular factor is small seen in relation to the seasonal factor. This means that the series contains a strong seasonal movement.

6.1.1.7 Retail index: Other consumption goods

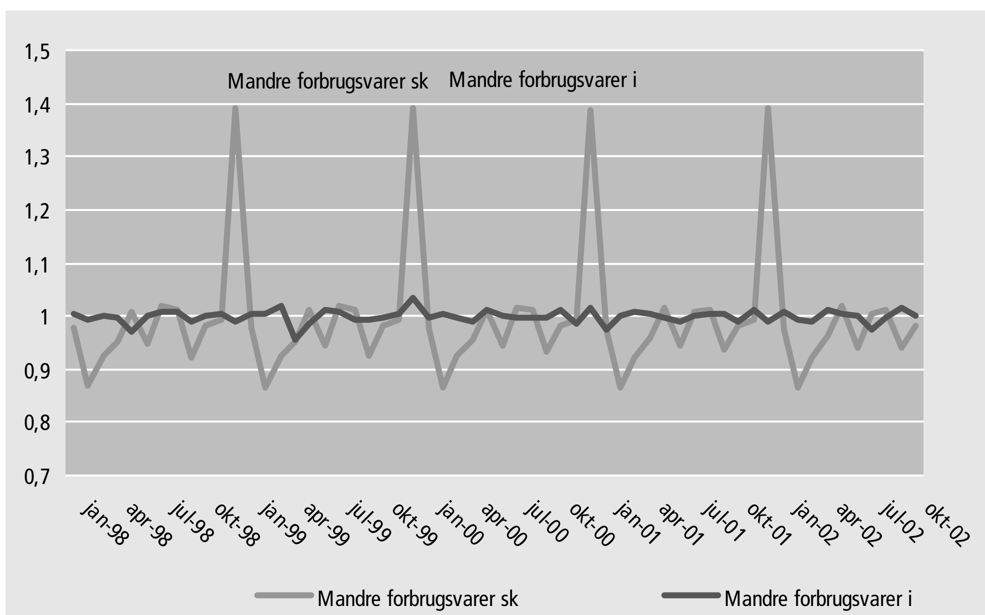
Graph 6.1.1.7.1



The result from the seasonal adjustment procedure shows that the series is indeed suitable for seasonal adjustment. Several test statistics have been tested, indicating the suitability of seasonal adjustment. All of these show that the series is performing well.

The series is monthly and is observed from January 1998 to October 2002, a multiplicative model has been chosen, and the possibility of an Easter effect has been tested. This effect is present in the series. The trading day effect has been tested as well and it is significant.

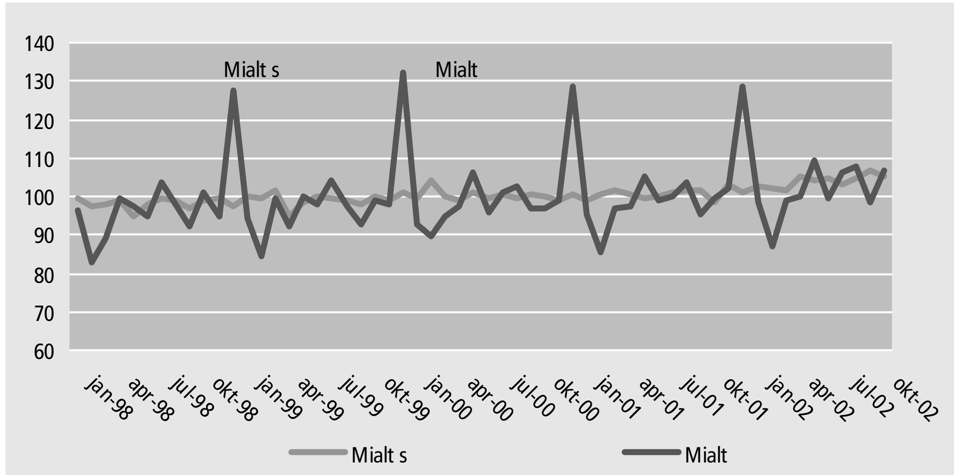
Graph 6.1.1.7.2



From the graph above, it is both seen that the estimated seasonal factor is stable over time, and that the irregular factor is small seen in relation to the seasonal factor. This means that the series contains a strong seasonal movement.

6.1.1.8 Retail index: Total

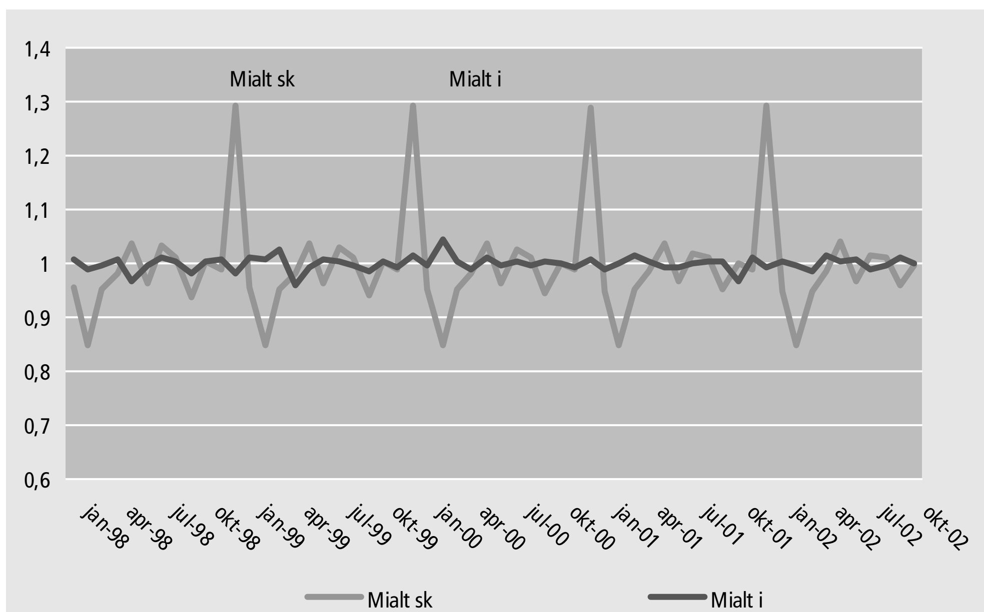
Graph 6.1.1.8.1



The result from the seasonal adjustment procedure shows that the series is indeed suitable for seasonal adjustment. Several test statistics have been calculated, indicating the suitability of seasonal adjustment. All of these show that the series is performing well.

The series is monthly and is observed from January 1998 to October 2002, a multiplicative model has been chosen, and the possibility of an Easter effect has been tested. This effect is present in the series. The trading day effect has been tested as well and it is significant.

Graph 6.1.1.8.2

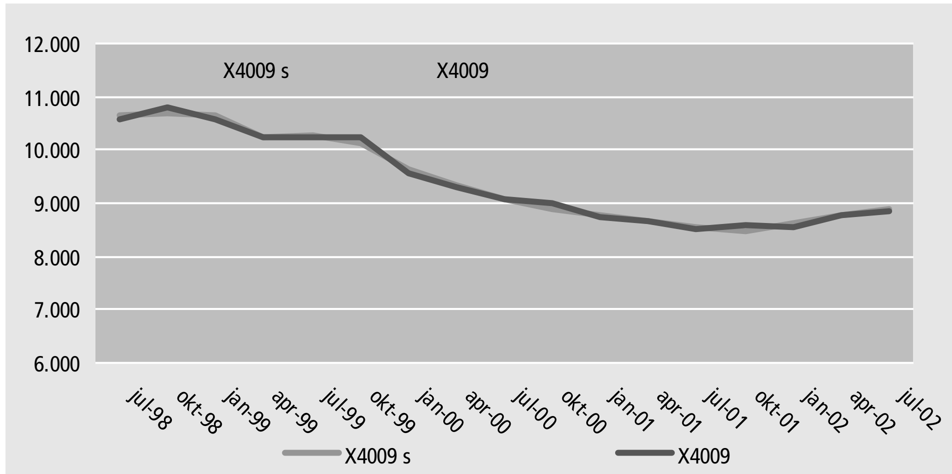


From the graph above, it is both seen that the estimated seasonal factor is stable over time, and that the irregular factor is small seen in relation to the seasonal factor. This means that the series contains a strong seasonal movement.

6.1.2 Series containing large irregular components

6.1.2.1 ATP-employment: Energy and water supply

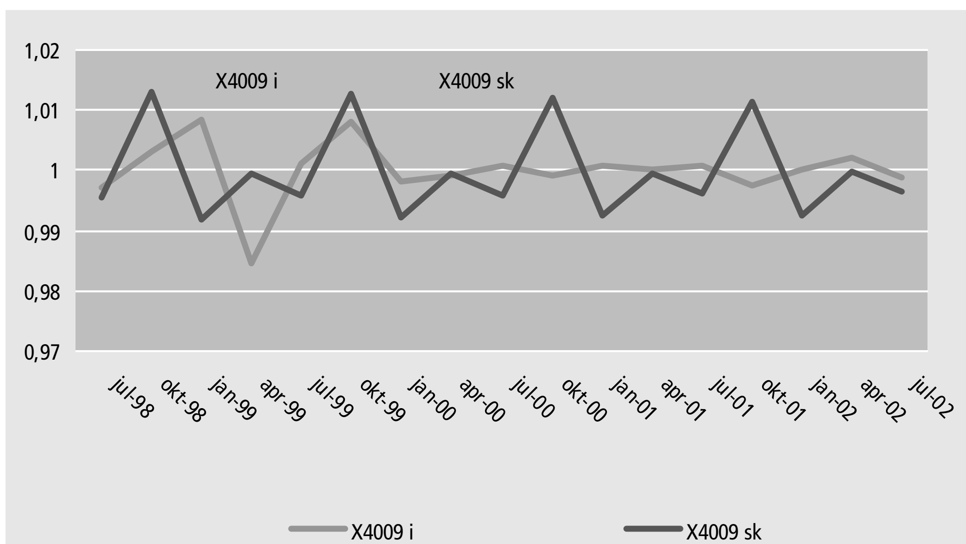
Graph 6.1.2.1.1



The result from the seasonal adjustment procedure shows that the series is indeed suitable for seasonal adjustment. Several test statistics have been tested, indicating the suitability of seasonal adjustment. Not all possible test statistics have been calculated because the series is so short. See table A.3.2 below for details.

The series is quarterly and is observed from 3q 1998 to 3q 2002, a multiplicative model has been chosen, and the possibility of an Easter effect has been tested. This effect is not present in the series. The trading day effect has not been tested, as this effect is irrelevant for the employment series.

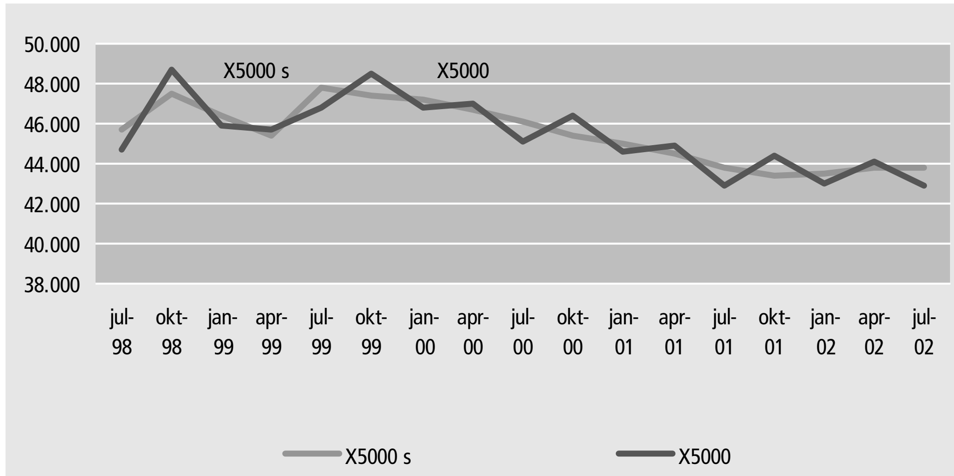
Graph 6.1.2.1.2



From the graph above, it is both seen that the estimated seasonal factor is stable over time, but the irregular factor sometimes dominates the seasonal factor. This means that the seasonal movement of the series from time to time is unclear as the irregular factor blurs the clear picture of the seasonal component.

6.1.2.2 ATP-employment: Trade, hotels and restaurants

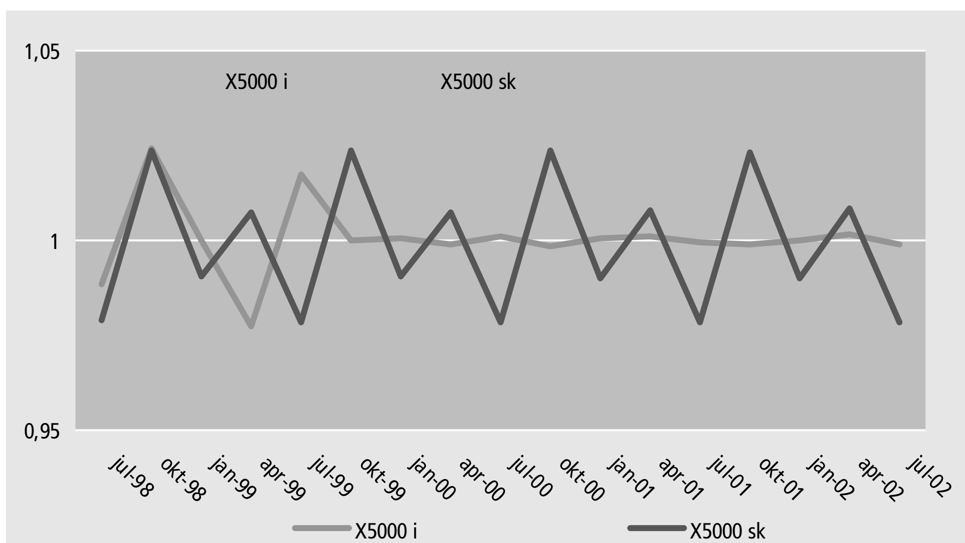
Graph 6.1.2.2.1



The result from the seasonal adjustment procedure shows that the series is indeed suitable for seasonal adjustment. Several test statistics have been calculated, indicating the suitability of seasonal adjustment. Not all possible test statistics have been calculated because the series is so short. See table A.3.2 below for details.

The series is quarterly and is observed from 3q 1998 to 3q 2002, a multiplicative model has been chosen, and the possibility of an Easter effect has been tested. This effect is not present in the series. The trading day effect has not been tested, as this effect is irrelevant for the employment series.

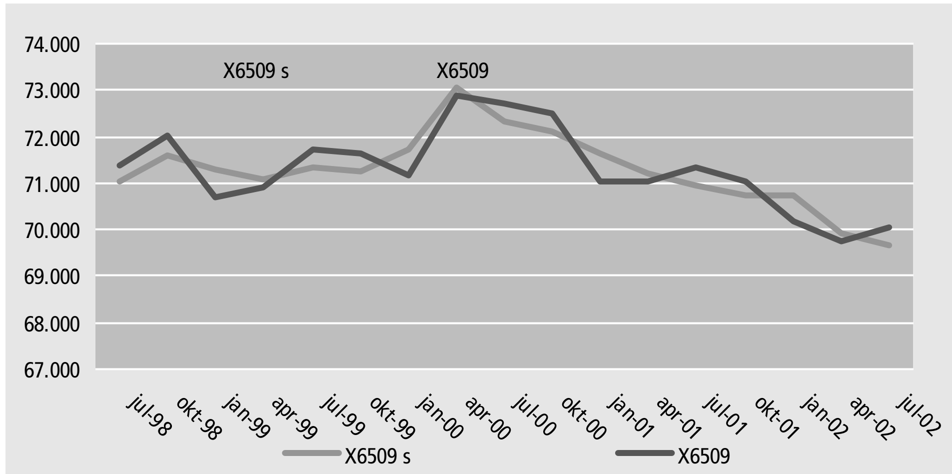
Graph 6.1.2.2.2



From the graph above, it is both seen that the estimated seasonal factor is stable over time, but the irregular factor sometimes dominates the seasonal factor. This means that the seasonal movement of the series from time to time is unclear as the irregular factor blurs the clear picture of the seasonal component.

6.1.2.3 ATP-employment: Finance

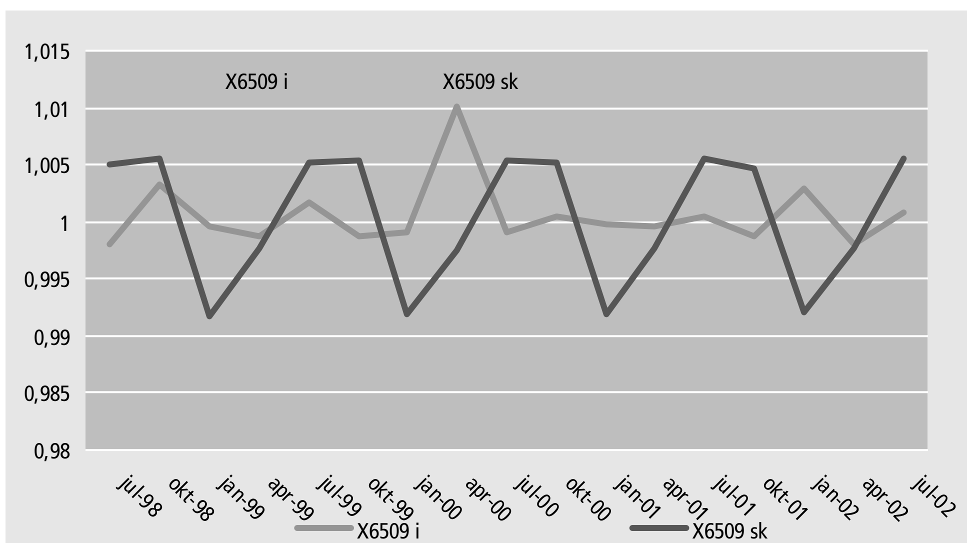
Graph 6.1.2.3.1



The result from the seasonal adjustment procedure shows that the series is indeed suitable for seasonal adjustment. There have been calculated several test statistics indicating the suitability of seasonal adjustment. Not all possible test statistics have been calculated because the series is so short. See the table A.3.2 below for details.

The series is quarterly and is observed from 3q 1998 to 3q 2002, a multiplicative model has been chosen, and there has been tested for the possibility of an Easter effect. This effect is not present in the series. The trading day effect has not been tested, as this effect is irrelevant for the employment series.

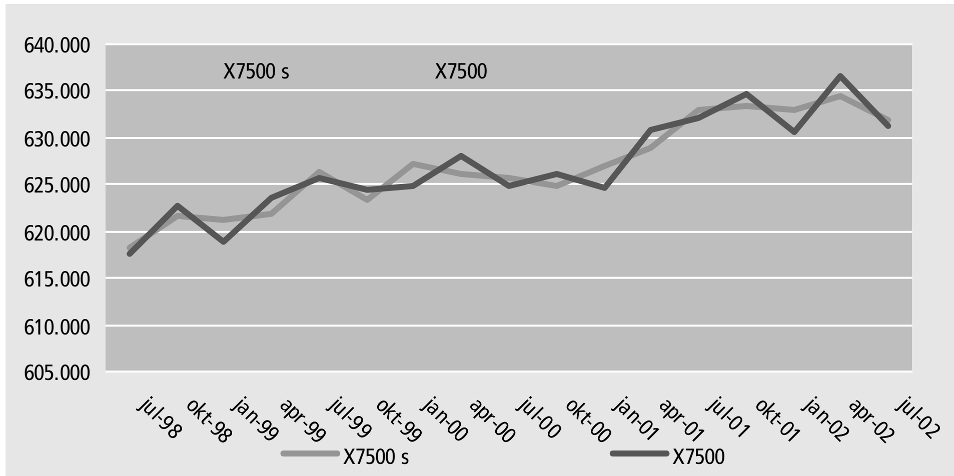
Graph 6.1.2.3.2



From the graph above, it is both seen that the estimated seasonal factor is stable over time, but the irregular factor sometimes dominates the seasonal factor. This means that the seasonal movement of the series from time to time is unclear as the irregular factor blurs the clear picture of the seasonal component.

6.1.2.4 ATP-employment: Public and personal services

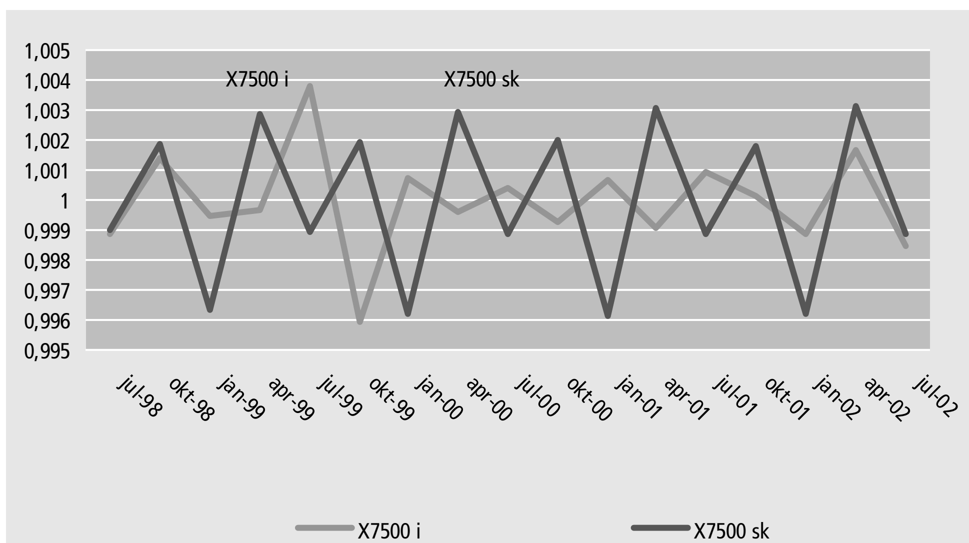
Graph 6.1.2.4.1



The result from the seasonal adjustment procedure shows that the series is indeed suitable for seasonal adjustment. Several test statistics have been calculated, indicating the suitability of seasonal adjustment. Not all possible test statistics have been calculated because the series is so short. See table A.3.2 below for details.

The series is quarterly and is observed from 3q 1998 to 3q 2002, a multiplicative model has been chosen, and the possibility of an Easter effect has been tested. This effect is not present in the series. The trading day effect has not been tested, as this effect is irrelevant for the employment series.

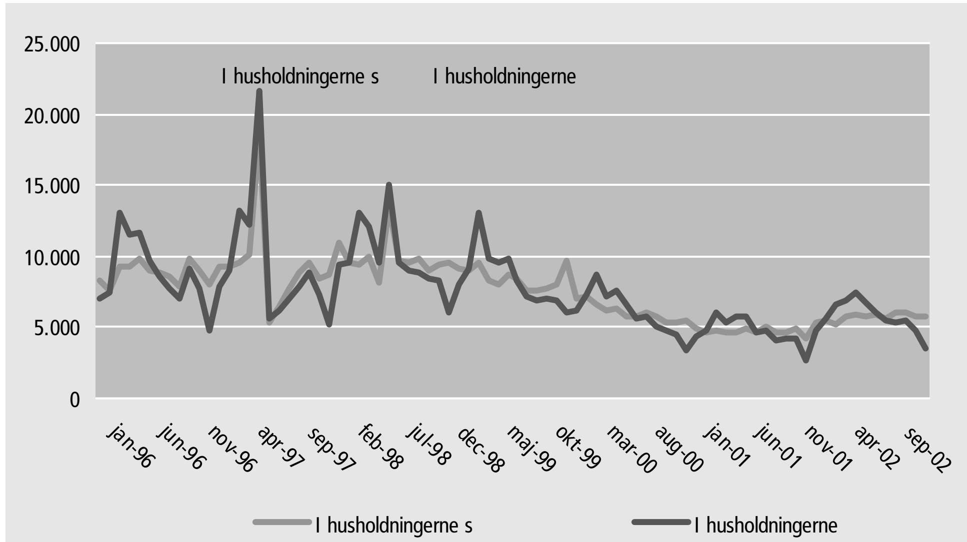
Graph 6.1.2.4.2



From the graph above, it is both seen that the estimated seasonal factor is stable over time, but the irregular factor sometimes dominates the seasonal factor. This means that the seasonal movement of the series from time to time is unclear as the irregular factor blurs the clear picture of the seasonal component.

6.1.2.5 Newly registered cars: Households

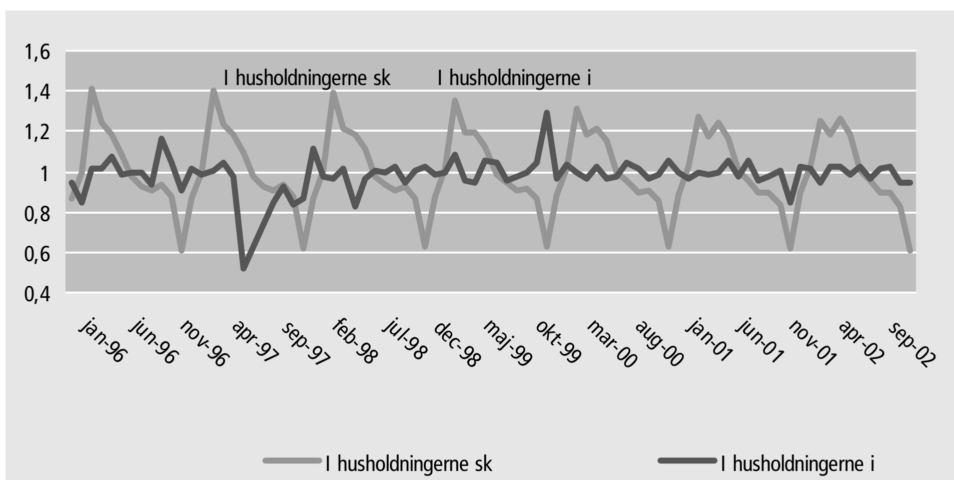
Graph 6.1.2.5.1



The result from the seasonal adjustment procedure shows that the series is indeed suitable for seasonal adjustment. Several test statistics have been tested, indicating the suitability of seasonal adjustment. All of these show that the series is performing well.

The series is monthly and is observed from January 1996 to December 2002, a multiplicative model has been chosen, and the possibility of an Easter effect has been tested. This effect is not present in the series. The trading day effect has been tested as well and neither this is significant.

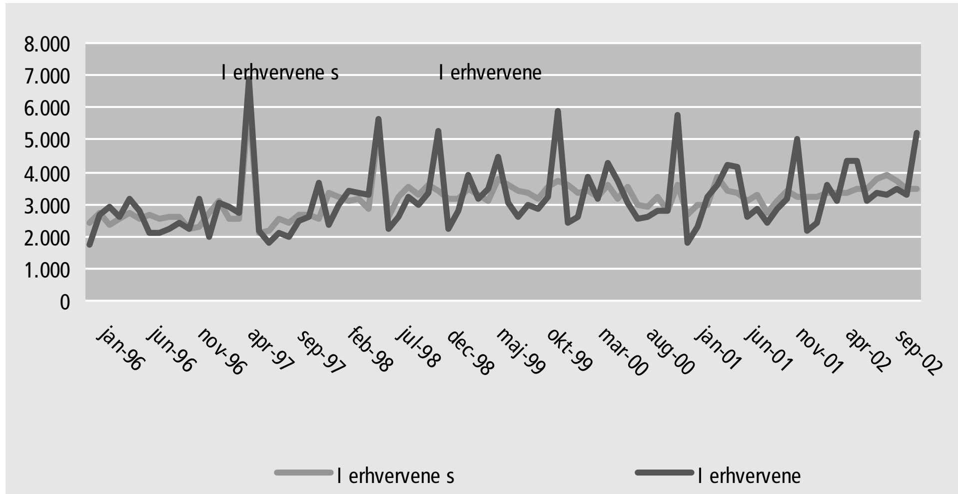
Graph 6.1.2.5.2



From the graph above, it is both seen that the estimated seasonal factor is stable over time, but the irregular factor sometimes dominates the seasonal factor. This means that the seasonal movement of the series from time to time is unclear as the irregular factor blurs the clear picture of the seasonal component.

6.1.2.6 Newly registered cars: Industry

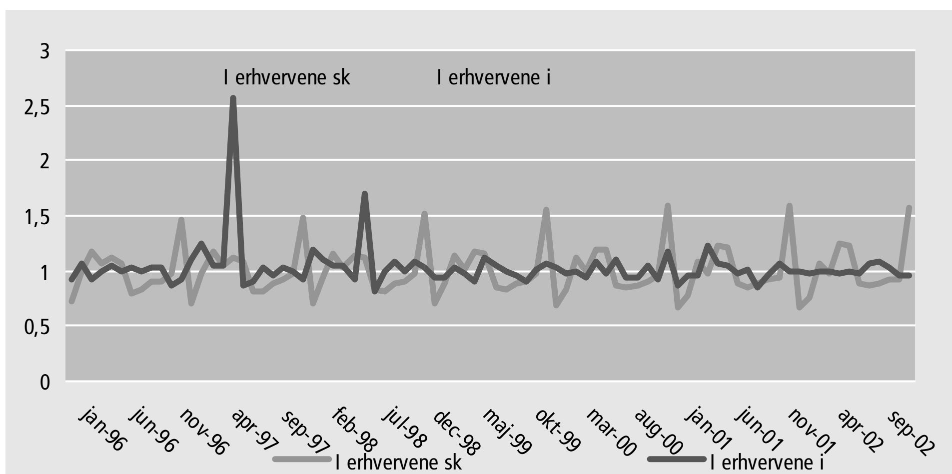
Graph 6.1.2.6.1



The result from the seasonal adjustment procedure shows that the series is indeed suitable for seasonal adjustment. Several test statistics have been tested, indicating the suitability of seasonal adjustment. All of these show that the series is performing well.

The series is monthly and is observed from January 1996 to December 2002, a multiplicative model has been chosen, and the possibility of an Easter effect has been tested. This effect is not present in the series. The trading day effect has been tested as well and this is significant.

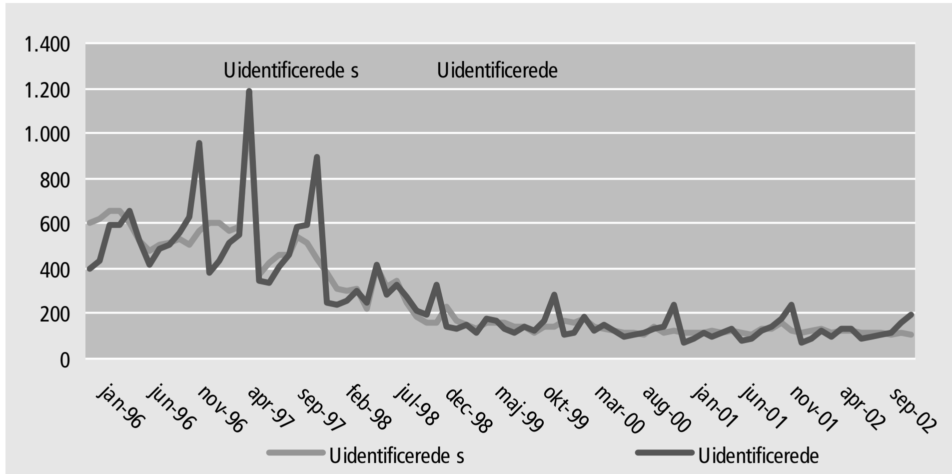
Graph 6.1.2.6.2



From the graph above, it is both seen that the estimated seasonal factor is stable over time, but the irregular factor sometimes dominates the seasonal factor. This means that the seasonal movement of the series from time to time is unclear as the irregular factor blurs the clear picture of the seasonal component.

6.1.2.7 Newly registered cars: Unidentified

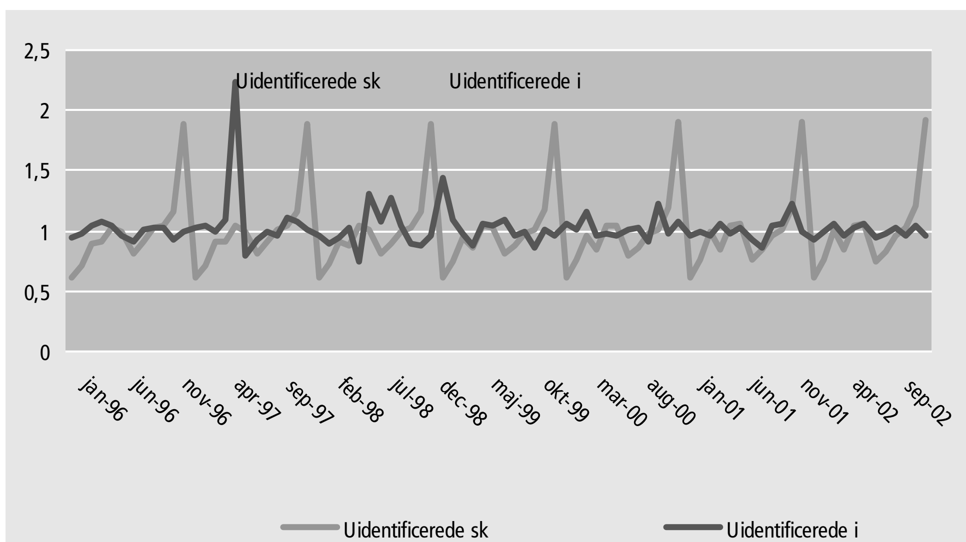
Graph 6.1.2.7.1



The result from the seasonal adjustment procedure shows that the series is indeed suitable for seasonal adjustment. Several test statistics have been calculated, indicating the suitability of seasonal adjustment. All of these show that the series is performing well.

The series is monthly and is observed from January 1996 to December 2002, a multiplicative model has been chosen, and the possibility of an Easter effect has been tested. This effect is not present in the series. The trading day effect has been tested as well and this is significant.

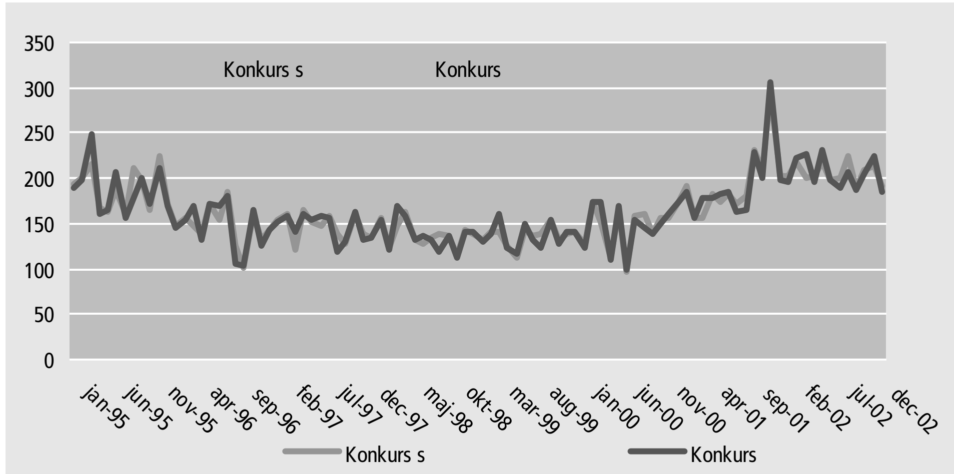
Graph 6.1.2.7.2



From the graph above, it is both seen that the estimated seasonal factor is stable over time, but the irregular factor sometimes dominates the seasonal factor. This means that the seasonal movement of the series from time to time is unclear as the irregular factor blurs the clear picture of the seasonal component.

6.1.2.8 Bankruptcies

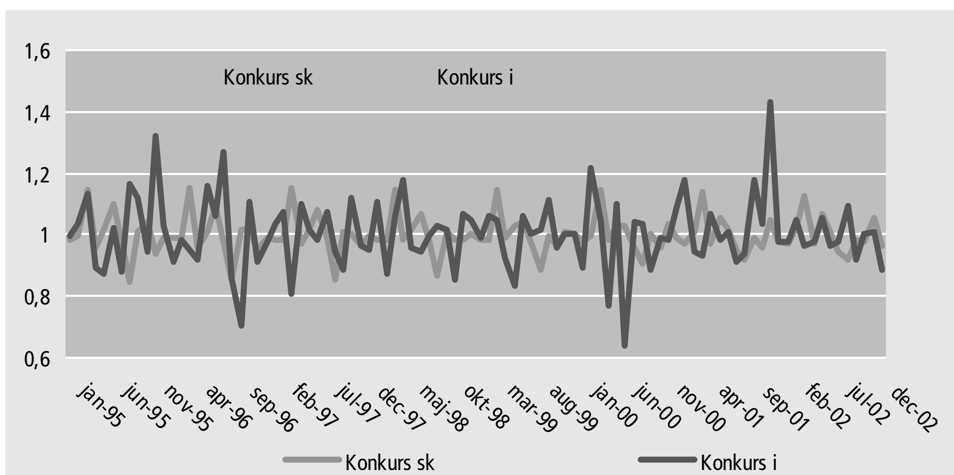
Graph 6.1.2.8.1



The result from the seasonal adjustment procedure shows that the series is indeed suitable for seasonal adjustment. Several test statistics have been calculated, indicating the suitability of seasonal adjustment. All of these show that the series is performing well.

The series is monthly and is observed from January 1995 to December 2002, a multiplicative model has been chosen, and the possibility of an Easter effect has been tested. This effect is not present in the series. The trading day effect has not been tested as this effect a priori is known not to be present in the series.

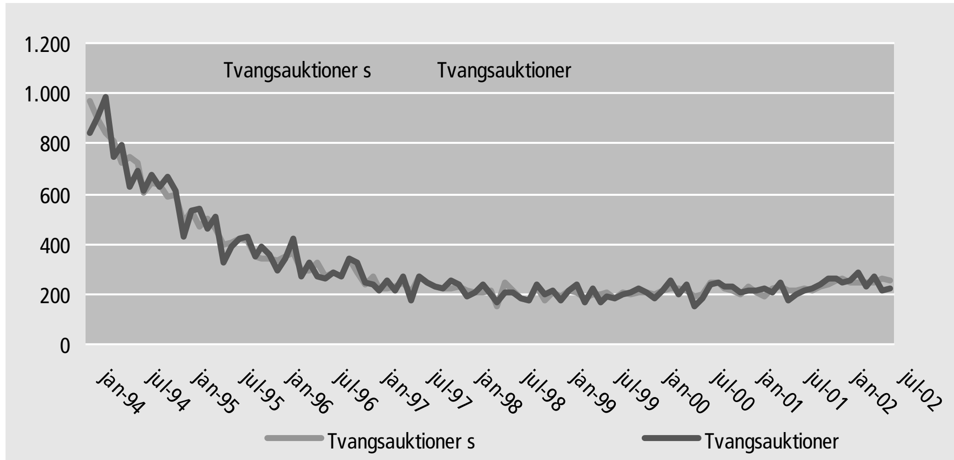
Graph 6.1.2.8.2



From the graph above, it is both seen that the estimated seasonal factor is stable over time, but the irregular factor sometimes dominates the seasonal factor. This means that the seasonal movement of the series from time to time is unclear as the irregular factor blurs the clear picture of the seasonal component.

6.1.2.9 Compulsory sales

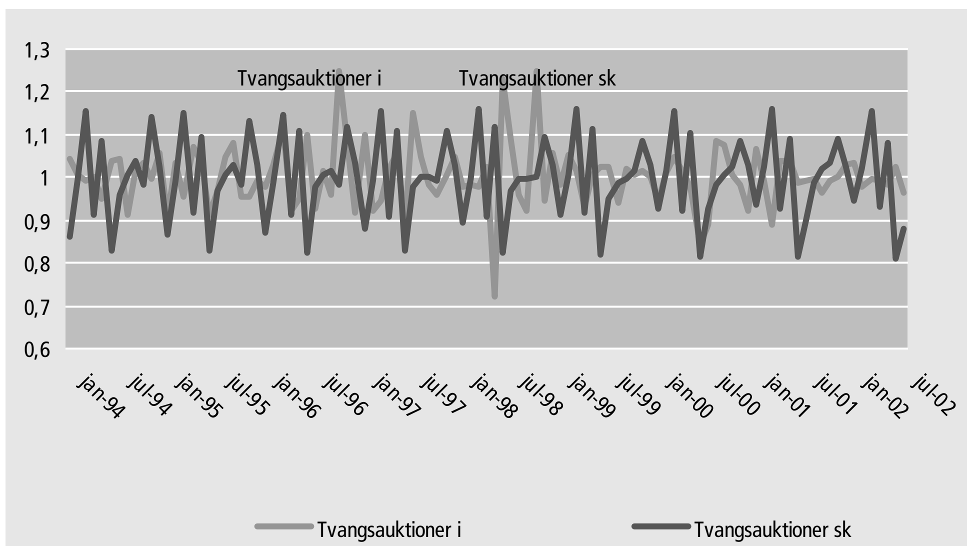
Graph 6.1.2.9.1



The result from the seasonal adjustment procedure shows that the series is indeed suitable for seasonal adjustment. Several test statistics have been calculated, indicating the suitability of seasonal adjustment. All of these show that the series is performing well.

The series is monthly and is observed from January 1994 to July 2002, a multiplicative model has been chosen, and the possibility of an Easter effect has been tested. This effect is not present in the series. The trading day effect has not been tested as this effect a priori is known not to be present in the series.

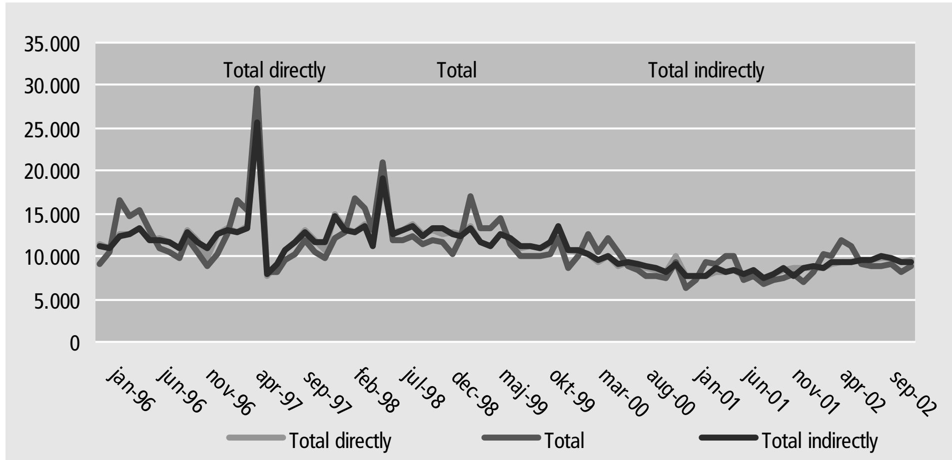
Graph 6.1.2.9.2



From the graph above, it is both seen that the estimated seasonal factor is stable over time, but the irregular factor sometimes dominates the seasonal factor. This means that the seasonal movement of the series from time to time is unclear as the irregular factor blurs the clear picture of the seasonal component.

6.1.3 Direct or indirect seasonal adjustment

6.1.3.1 Newly registered cars, total



The series called Newly registered cars, total has been seasonally adjusted both using the direct approach and the indirect approach. The original series together with the seasonal adjusted series in both approaches are shown in the figure above.

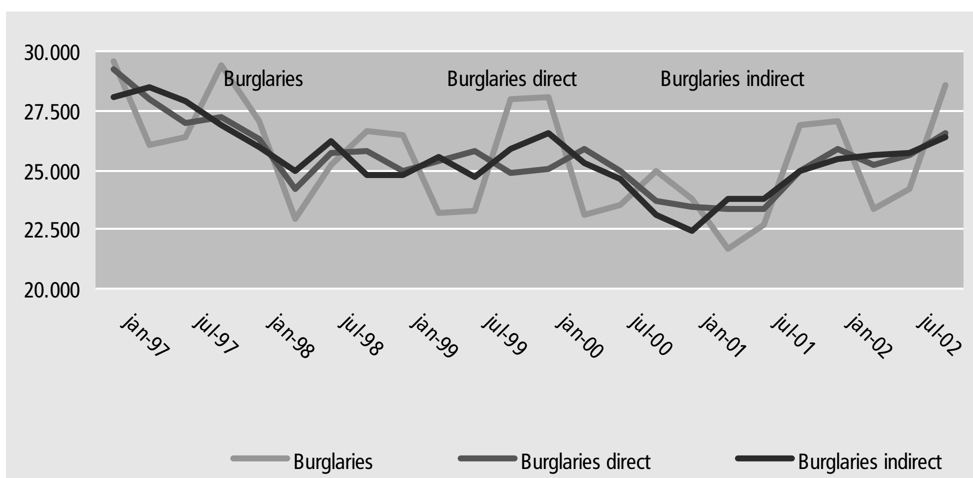
The total series is a sum of the following three series:

- Newly registered cars in households
- Newly registered cars in industry
- Newly registered cars unidentified

The directly seasonally adjusted series is the seasonally adjusted aggregated series. The indirectly seasonally adjusted series is the sum of the seasonally adjusted component series.

From the graph, it is seen that these two seasonally adjusted series are basically equal.

6.1.3.2 Burglaries, total



The series called burglaries, total has been seasonally adjusted both using the direct approach and the indirect approach. The original series together with the seasonal adjusted series in both approaches are shown in the figure above.

The total series is a sum of the following three series:

- Burglaries in banks
- burglaries in houses
- burglaries in weekend cottages

The directly seasonally adjusted series is the seasonally adjusted aggregated series. The indirectly seasonally adjusted series is the sum of the seasonally adjusted component series.

From the graph, it is seen that these two seasonally adjusted series are basically equal.

Appendix 1: Moving Averages

A1.1 Symmetric moving averages

A centred moving average is calculated over a number of elements of a time series. Denote this series $x_1, \ldots, x_n$, where $n \geq 1$. Let the *x*'s be a time series with monthly observations. A three-month long centred moving average has the following form:

$$\frac{x_{t-1} + x_t + x_{t+1}}{3}, \text{ for } 2 \leq t \leq n-1.$$

A four-month long centred moving average has the following form:

$$\frac{\frac{1}{2}x_{t-2} + x_{t-1} + x_t + x_{t+1} + \frac{1}{2}x_{t+2}}{4}, \text{ for } 3 \leq t \leq n-2.$$

From the formulas given above, a general feature is seen, namely that the centred m-months long moving averages are simple whenever m is odd, and the formula turns more complicated whenever m is even. The general formula for an m-month long moving average is given below:

An m-period long moving average, when m is odd, is given as follows;

$$\frac{1}{m}\sum_{j=-(\frac{m-1}{2})}^{\frac{m-1}{2}} x_{t+j}, \text{ for } 1+\frac{m-1}{2} \leq t \leq n-\frac{m-1}{2}$$

An m-period long moving average, when m is even, is given as follows;

$$\frac{1}{m}\left[\frac{1}{2}(x_{t-\frac{m}{2}} + x_{t+\frac{m}{2}}) + \sum_{j=1-\frac{m}{2}}^{\frac{m}{2}-1} x_{t+j}\right], \text{ for } 1+\frac{m}{2} \leq t \leq n-\frac{m}{2}$$

Often moving averages of the so-called type n×m are given. The general form of this type of moving average will not be given here, but it will only be shown in two special cases, namely a 3×3-moving average and a 2×4 moving average is given below.
Both these types of moving averages have five terms as is seen from the formulas below;

$$\bar{S}_t^{3\times3} = \frac{1}{3}\left[\frac{s_{t-2} + s_{t-1} + s_t}{3} + \frac{s_{t-1} + s_t + s_{t+1}}{3} + \frac{s_t + s_{t+1} + s_{t+2}}{3}\right]$$

$$= \frac{1}{9}s_{t-2} + \frac{2}{9}s_{t-1} + \frac{1}{3}s_t + \frac{2}{9}s_{t+1} + \frac{1}{9}s_{t+2}$$

$$\bar{S}_t^{2\times4} = \frac{1}{2}\left[\frac{s_{t-2}+s_{t-1}+s_t+s_{t+1}}{4}+\frac{s_{t-1}+s_t+s_{t+1}+s_{t+2}}{4}\right]$$

$$=\frac{1}{8}s_{t-2}+\frac{1}{4}s_{t-1}+\frac{1}{4}s_t+\frac{1}{4}s_{t+1}+\frac{1}{8}s_{t+2}$$

A1.2 Henderson's ideal formula

An often applied form of the moving average is the so-called Henderson moving average where the weights are calculated according to the Henderson ideal formula. Henderson's formula is ideal in the sense that the squared sum of the 3rd difference between the Henderson Moving Averages (HMA) weights is minimized. Let k be the length of the moving average, and the two variables z and m are given as k=2z-3=2m+1. Then

$$kHMA(x_t)=\sum_{i=t-m}^{t+m}\frac{315[(z-1)^2-i^2][z^2-i^2][(z+1)^2-i^2][(3z^2-16)-11i^2]}{8z(z^2-1)(4z^2-1)(4z^2-9)(4z^2-25)}x_i$$

A1.3 Comparing MA-weights

When the aim in the seasonal adjustment procedure is to smoothen observations, then there exists no optimal type of moving average, neither exists an optimal length of the applied average. The choice will depend on the magnitude of the amplitudes of the series and obviously of the goal of the analysis. In general terms, it can be said that the longer ´the moving average, the smoother the result series. The smoothening of a time series also depends on the chosen weight structure. Below, different weights are calculated for different types of moving averages of five periods.

Table A.1.3.1

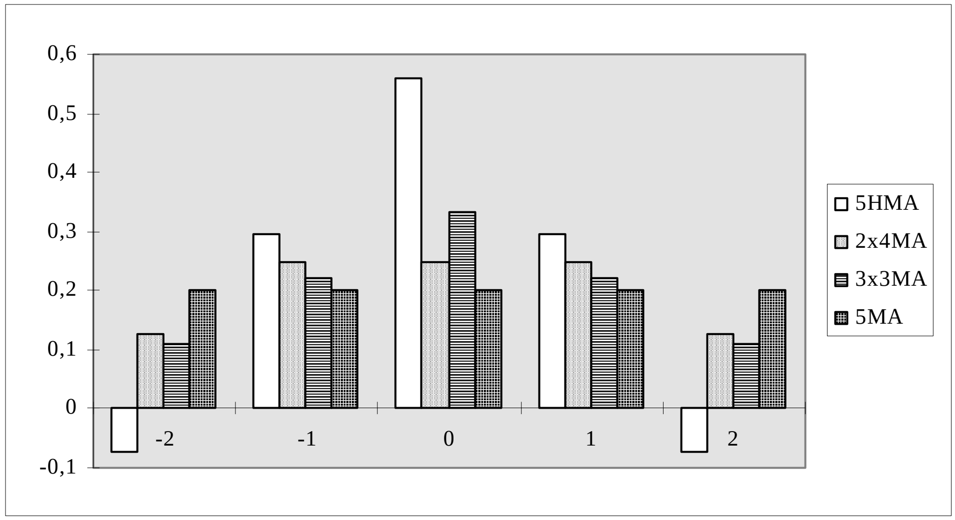
Weights for different types of five-period long moving averages.

Periods i	5HMA	2x4MA	3x3MA	5M A
-2	-0,073	0,125	0,111	0,2
-1	0,294	0,250	0,222	0,2
0	0,559	0,250	0,333	0,2
1	0,294	0,250	0,222	0,2
2	-0,073	0,125	0,111	0,2

The weights of the Henderson average with 5 terms can be found in the formula of appendix A.1.2 where k=5 yields that z=4 and m=2. Therefore, the weights for i=-2,-1,0,1,2 are given as

$$\frac{315\left(9-i^2\right)\left(16-i^2\right)\left(25-i^2\right)\left(32-11i^2\right)}{32\times15\times63\times55\times39}$$

By comparing the weights of these four types of averages graphically, it is seen that a Henderson moving average assigns a high weight to the observations around the centre. Therefore, this type of moving average is not as smoothening as the other types of averages in the example.



Appendix 2: The Q-test statistic in the X-12 procedure

The main indicator for the quality of the seasonal adjustment is the so-called Q-test statistic. This test statistic is calculated as a weighted average of 11 different component test statistics called M1,…,M11. Q is calculated as follows;

Weights used in the calculation of Q when the series is longer than 6 years.

M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11
13	13	10	5	11	10	16	7	7	4	4

Weights used in the calculation of Q when the series is shorter than 6 years.

M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11
17	17	10	5	11	10	30	0	0	0	0

It should be noticed that the weights in the second table is not a simple rescaling of the weights in the first table.

All test statistics are standardized so that they range between 0 and 3, and they have been normalized so that the acceptance area is the interval [0;1].

M1

The seasonal component and the irregular component cannot be separated suitably if the variation of the irregular component is too high when they are compared to the variation of the seasonal component. M1 measures the relative contribution (to the variance of the percentage wise changes in the components of the original series) from the irregular component over a three-month long interval.

If the contribution from the irregular component is too large, it can be interpreted to be due to the circumstance that the variation of the irregular component dominates the variation of the seasonal component.

Define now $\bar{I}^2 = \frac{1}{n-1}\sum_{t=2}^{n}(I_t - I_{t-1})^2$, where I_t Are the final estimated irregular factors, and n is the number of observations in the series. Using the same formula $\bar{C}$ and $\bar{S}$ is defined. Consider now the fraction

$$\frac{\bar{I}^2}{\bar{I}^2 + \bar{C}^2 + \bar{S}^2}$$

If this fraction exceeds 0.1 then the series is considered behaving unsatisfactorily with respect to the variation of the irregular factor seen in relation to the overall variation.

The fraction

$$M1 = \frac{\bar{I}^2}{\bar{I}^2 + \bar{C}^2 + \bar{S}^2} \times 10\ .$$

has to be smaller than 1.

M2

M2 Is equivalent to M1. The only difference is the procedure when removing the trend (in order to make the series stationary). A line is fitted to the trend-cycle estimates in order to obtain a trend estimate. This trend estimate is removed from the original series and hence a stationary original series is obtained, this series is denoted B1'. The finally estimated trend-cycle component is de-trended in the same manner. Denote this de-trended series D12'. Hereafter, a fraction corresponding to the contribution from the irregular component is calculated. If this test statistic exceeds 1, then the hypothesis of the non-dominating irregular component is rejected.

M3

In the test statistics M3, a comparison between the magnitude of the monthly/quarterly change in the irregular component is compared to the magnitude of the monthly/quarterly change in the trend-cycle. The aim of the seasonal adjustment is removing the seasonal component from the original series in order to give an estimate of a seasonally adjusted series. As X-12 is an iterative process, it is mainly important that not only is the seasonal factor estimated thoroughly throughout the steps that lead to the final seasonal adjustment, but also the trend-cycle and the irregular component have to be estimated thoroughly. If the movement of the irregular factor from one period to another is dominating in the CI-series, then it is difficult to separate these two components and the quality of the seasonal adjustment is therefore low.
The so-called $\tilde{I} / \tilde{C}$ Rate is therefore calculated;

$$\tilde{I} / \tilde{C} = \frac{\sum_{t=2}^{n} \left| I_t - I_{t-1} \right| / I_{t-1}}{\sum_{t=2}^{n} \left| C_t - C_{t-1} \right| / C_{t-1}}$$

If the $\tilde{I} / \tilde{C}$ rate is high then the variation in the seasonally adjusted series is only due to the irregular component. The rate is considered high if it exceeds 3, and the corresponding test statistic for monthly series is therefore:

$$M3 = \left| \tilde{I} / \tilde{C} - 1 \right| / 2$$

And for quarterly series it equals:

$$M3 = \left| \tilde{I} / \tilde{C} - \frac{1}{3} \right| / \frac{2}{3}$$

M4

The test statistic M4 examines the extent of 1st order autocorrelation in the irregular component. A basic assumption in the F-test statistics in X-12 is that the irregular component is a series of white noise. That is a series of independently identically distributed error terms with mean 0, a constant variance and the covariance between these terms are 0. X-12 uses a so-called sign-test denoted ADR (Average Duration of Run) in order to test the randomness in the finally estimated residuals.

This non-parametric test is based on the number of turning points. The test is designed to test the randomness in the estimated residuals against a hypothesis saying that the error terms are a AR(1)-process. For a white noise process (with an infinite number of observations) the ADR equals 1.5. The M4 test statistic is based on the formula by Bradley on approximation of the normal distribution:

$$M4 = \frac{\left|\frac{n-1}{ADR} - \frac{2(n-1)}{3}\right|}{\sqrt{\frac{16n-29}{90}}} \times \frac{1}{2{,}58}$$

where 2.58 is the 1% significance level in a two-sided normal distribution test. If M4 exceeds 1 then autocorrelation is present in the residuals.

M5

M5 is an indication of the number of periods that it takes the average absolute changes in the trend-cycle to dominate the corresponding change in the irregular component. The test statistic is equivalent to M3 in the sense that it examines the relative magnitude of the changes in the irregular factor and the trend-cycle components. For k=1,...,12 (or k=1,2,3,4 for quarterly series) the fraction

$$\frac{\frac{1}{n-1}\sum_{j=k+1}^{n}(I_t - I_{t-k})^2}{\frac{1}{n-1}\sum_{j=k+1}^{n}(C_t - C_{t-k})^2} \equiv \frac{\bar{I}(k)}{\bar{C}(k)} .$$

is calculated. Thereafter, the Monthly Cyclical Dominance (MCD) is derived (for quarterly series QCD is derived) as

$$\bar{I}(k)/\bar{C}(k) \leq 1 \wedge \bar{I}(k-1)/\bar{C}(k-1) > 1 \Rightarrow MCD = k$$

MCD only takes integer values. It is constructed to be rather ‘silly’ since it does not notice how close to 1 the I/C ratio is. In order to make MCD 'wiser', a new test statistic MCD' has been constructed in order to solve this problem.

MCD' interpolates linearly the I/C-ratio in order to determine its equalization to 1. The exact formula for MCD' will not be given, but it has to be noticed that whenever MCD' takes a value exceeding 6 then it is not acceptable. Therefore, M5 has the form:

$$M5 = \frac{|MCD' - 0,5|}{5}$$ for monthly series, and

$$M5 = \frac{|QCD' - 0,17|}{1,67}$$ for quarterly series.

Therefore, it holds that M5>1 when MCD's or QCD's are unacceptably high.

M6

In the test statistic M6, the magnitude of the yearly changes in the irregular factor is compared to the magnitude of the yearly change in the seasonal factor. When the aim of the seasonal adjustment is taken into consideration, it is seen that it is very important to be able to identify the seasonal factors. In order to separate the irregular factor from the seasonal component X-12 uses a 3 x 5 moving average on the SI-ratio.
Experiences have shown though, that if the yearly change in the irregular factor is too small (compared to the yearly change in the seasonal component), that is when the I/S-ratio is low, then at 3 x 5 is insufficient when it comes to following the seasonal movement. On the other hand, the 3 x 5 seasonal filter is too flexible when the I/S-ratio is too high, and the seasonal factors that have been determined contain some of the irregular movement. Empirical studies have shown that the 3 x 5 moving average is superior when the I/S-ratio lies in the interval [1,56,5]. Therefore

$$M6 = \frac{|\bar{I} / \bar{S} - 4|}{2,5}$$

Whenever M6 is larger than 1 the hypothesis is rejected, but the problem of a high value of M6 can be solved by using a 3x1 moving average whenever the I/S-ratio is smaller than 1,5, or by using the stable seasonal pattern (stable seasonality option) of the relation exceeds 6,5.

M7

M7 tests the degree of stable seasonality seen in relation to the degree of movable seasonality. If a time series that has been corrected for the trend-cycle (the SI-rate) only exhibits a little amount of stable seasonality seen in relation to the movable seasonality, then the identification of the stable seasonal variation is difficult. The test that is applied is a combination of the F-test applied on the final SI-ratios from X-11, and a test statistic that has been developed by Statistics Canada by J. Higginson (1975). This test indicates whether the seasonal pattern is identifiable by X-12 or not. The test from X-11 measures the magnitude of stable seasonal pattern in the time series. Den0ote this test statistic F_s , while Higgins' test examines whether movable seasonality is present in the series. Denote this test statistic F_m. The seasonal pattern is identifiable if the absolute error that is present in the final estimates for the seasonal factors is small. This error depends on both the F-test statistics mentioned above. If F_s Is low then a high degree of disturbance is found and if furthermore F_m is high, more disturbances are to be found in the series, namely the disturbance due to the movable seasonality.

M7 is hence a combination of the two F-tests mentioned above.

$$M7 = \sqrt{\frac{1}{2}(\frac{7}{F_s} + 3\frac{F_m}{F_s})}$$

About M8,..., M11

In the full length of a time series, only a constant seasonal component can be optimally estimated. This is due to the seasonal filters that are applied in X-12. As such the estimates of the seasonal factors contain a considerable error if the original series contains year-to-year movements. Two types of movements are considered quite differently. That is, the ones exhibiting random fluctuations and the ones where changes prevail in the sane direction throughout the years.

The magnitude of the first-mentioned movement can be measured, using the average year-to-year change in the seasonal factors. The magnitude of the second-mentioned movement can be measured by a simple arithmetic average of the changes. Such an average will namely give an indication of the magnitude of the systematic (linear) movement.

Random movements are measured by the test statistics M8 and M10. The test statistics M9 and M11 describe the magnitude of the linear movement. M8 and M10 use all data in the calculations, while M10 and M11 are calculated only on the basis of data from the latest periods.

The test statistics M10 and M11 were introduced since users of seasonally adjusted figures only are focused on the quality of the seasonal adjustment in the latest available years, and these test statistics concentrate only on the seasonal movement in the end of the series.

It is especially important to know whether there is an unambiguously determined linear movement in the seasonal factors for the most recent years. If this linearity is present then the estimates for the seasonal factors are disturbed significantly by the seasonal filters. It is the same disturbance that prevents the use of the seasonal factors from the most recent years when measuring the magnitude of the seasonal movement. This problem is solved by examining the three years before the most recent three year period. It is hoped that the seasonal movement remains the same in these new final years.

The test statistics M8,...,M11 are based upon the normalized seasonal factors:

$$S'_t = \frac{S_t - \bar{S}}{\sqrt{\frac{1}{n-1}\sum_{t=1}^{n}(S_t - \bar{S})^2}}$$

M8

The test statistic M8 measures the random fluctuation of the seasonal factors in the full range of the series. A high value indicates a high degree in the X-12 estimation of the seasonal factors. If the seasonal factors for each year are very different (and random) then the seasonal adjustment is not usable as a very unstable seasonal pattern has been determined.

The variation in the seasonal factors can be measured by the following;

$$\left|\Delta\bar{S}'\right| = \frac{1}{m(T-1)}\sum_{j=1}^{m}\sum_{i=2}^{T}\left|S'_{mi+j} - S'_{m(i-1)+j}\right|$$

where m is the number of observations within a calendar year (I.e. either 4 or 12), and T is the number of years.

As the average acceptable value for the variation in the seasonal factors is set equal to 10 pct., the test statistic M8 is the following:

$$M8 = 10\left|\Delta\bar{S}'\right|$$

In the calculation of M8, data are only used from the years where the seasonal factors have been calculated without using extrapolation.

M9

The test statistic is used for testing the average linear movement in the seasonal factors in the full length period. When an average of the year to year changes is formed, the amount of systematic movement in the series is measured. If the only present fluctuations are random then this average will be close to zero. If most changes occur in the same direction then the mean absolute change is close to the average arithmetic change.

As it holds that (telescopic sum) $\sum_{i=1}^{n-1}\Delta S'_{mi+j} = S'_{m(n-1)} - S'_j$, and an acceptance limit of 0,1,it holds that:

$$M9 = 100 \times \frac{\sum_{j=1}^{m}\left|S'_{m(n-1)+j} - S'_j\right|}{m(n-1)} \times \frac{1}{10}$$

M10 and M11

The test statistics are identical to M8 and M9, respectively, but are only calculated using the years n-2, n-3, n-4 and n-5. Users are typically often only interested in the latest available data, and this is the reason for these test statistics to provide information on the quality of the latest estimates of the seasonal factors. In the calculations, data are only used from the most recent years where the seasonal factors have been calculated without the use of extrapolations.

Appendix 3: Model specifications and test statistics for the series

Table A.3.1 **Model specifications**

Series	Fre-quency	Model	Start	End	ARIMA-model	Easter effect	Trading day effect
Newly registered cars							
In households	m	M	Jan 96	Dec 2002	(0 1 1)(0 1 1)	No	No
In industry	m	M	Jan 96	Dec 2002	(0 1 1)(0 1 1)	No	Yes
Unidentified	m	M	Jan 96	Dec 2002	(0 1 1)(0 1 1)	No	Yes
Retail index							
Food and beverages	m	M	Jan 98	Oct 2002	(0 1 1)(0 1 1)	Yes	Yes
Clothing	m	M	Jan 98	Oct 2002	(0 1 1)(0 1 1)	No	Yes
Other consumption goods	m	M	Jan 98	Oct 2002	(0 1 1)(0 1 1)	Yes	Yes
Total	m	M	Jan 98	Oct 2002	(0 1 1)(0 1 1)	Yes	Yes
Compulsory sales and Bankruptcies							
Declared Bankruptcies	m	M	Jan 95	Dec 2002	(0 1 1)(0 1 1)	No	No
Proclaimed compulsory sales	m	M	Jan 94	July 2002	(0 1 1)(0 1 1)	No	Yes
Employment figures on the basis of ATP payments							
Agriculture	q	M	3q 1998	3q 2002	(2 1 1)(0 1 1)	No	No
Industry	q	M	3q 1998	3q 2002	(3 1 0)(0 1 1)	No	No
Energy- and water supply	q	M	3q 1998	3q 2002	(3 1 1)(0 1 1)	No	No
Construction	q	M	3q 1998	3q 2002	(3 1 1)(0 1 1)	No	No
Trade, hotels and restaurants	q	M	3q 1998	3q 2002	(3 1 1)(0 1 1)	No	No
Transportation, mail and telecommunication	q	M	3q 1998	3q 2002	(0 1 1)(0 1 1)	No	No
Finance	q	M	3q 1998	3q 2002	(3 1 1)(0 1 1)	No	No
Public and personal services	q	M	3q 1998	3q 2002	(0 1 1)(0 1 1)	No	No

Table A.3.2 **Test statistics**

Series	SA quality index	Ljung-Box on residuals	Kurtotis	Forecast error over last year	Percentage of outliers	Q-test statistic
Newly registered cars						
In households	8,728 ([0;10])	35,8 ([0;51,20])	2,32 ([1,24;4,76])	32,10% ([0%;15%])	3,57% ([0%;5%])	0,42 ([0;1])
In industry	5,069 ([0;10])	13,72 ([0;51,20])	2,58 ([1,20;4,80])	10,16% ([0%;15%])	2,50% ([0%;5%])	0,86 ([0;1])
Unidentified	3,964 ([0;10])	31,47 ([0;51,20])	3,04 ([1,22;4,78])	10,30% ([0%;15%])	1,22% ([0%;5%])	0,42 ([0;1])
Retail index						
Food and beverages	4,037 ([0;10])	28,54 ([0;51,20])	-	1,70% ([0%;15%])	0% ([0%;5%])	0,94 ([0;1])
Clothing	2,660 ([0;10])	14,16 ([0;51,20])	-	3,54% ([0%;15%])	0% ([0%;5%])	0.55 ([0;1])
Other consumption goods	2,820 ([0;10])	21,32 ([0;51,20])	-	2,59% ([0%;15%])	0% ([0%;5%])	0.54 ([0;1])
Total	3,812 ([0;10])	27,11 ([0;51,20])	-	2,06% ([0%;15%])	0% ([0%;5%])	0.86 ([0;1])
Compulsory sales and Bankruptcies						
Declared Bankruptcies	7,229 ([0;10])	27,97 ([0;51,20])	3,53 ([1,35;4,65])	13,35% ([0%;15%])	0% ([0%;5%])	1,86 ([0;1])
Proclaimed compulsory sales	3,669 ([0;10])	26,80 ([0;51,20])	3,05 ([1,40;4,60])	5,67% ([0%;15%])	0% ([0%;5%])	0,90 ([0;1])
Employment figures on the basis of ATP payments						
Agriculture	1,144 ([0;10])	-	-	0,64% ([0%;15%])	0% ([0%;5%])	-
Industry	1,384 ([0;10])	-	-	3,27% ([0%;15%])	0% ([0%;5%])	-
Energy- and water supply	2,008 ([0;10])	-	-	4,23% ([0%;15%])	0% ([0%;5%])	-
onstruction	1,828			1,64%	0%	

	([0;10])	-		([0%;15%])	([0%;5%])	
Transportation, mail and telecommunication	1,901 ([0;10])	-	-	0,56% ([0%;15%])	0% ([0%;5%])	-
Finance	1,720 ([0;10])	-	-	0,33% ([0%;15%])	0% ([0%;5%])	-
Public and personal services	2,654 ([0;10])	-	-	0,26% ([0%;15%])	0% ([0%;5%])	-